CHAPTER 2

In the practice of numerical analysis it is important to be aware that computed solutions are not exact mathematical solutions. Perfect accuracy in most computational processes is impossible. We must make certain approximations, and this introduced **errors**.

The **error** in a computed quantity is defined as

$$\text{Error} = \text{true value} - \text{approximate value}$$
$$= x_T - x_A$$

The **relative error** is a measure of the error in relation to the size of the true being sought:

$$\text{Relative error} = \left|\frac{\text{error}}{\text{true value}}\right|$$

$$= \left|\frac{x_T - x_A}{x_T}\right|$$

$$\text{True percent relative error}, \varepsilon_t = \frac{\text{true value} - \text{approximate value}}{\text{true value}} \times 100\%$$

$$\text{Approximate percent relative error}, \varepsilon_a = \frac{\text{present approximation} - \text{previous approximation}}{\text{present approximation}} \times 100\%$$

Round-off error

Example :

Numbers such as π, e, or $\sqrt{3}$ can not be expressed by a fixed number of decimal places. Therefore they can not be represented exactly by the computer.

Consider the number π. It is irrational, i.e. it has infinitely many digits after the period:

$$\pi = 3.14159265358979323846264338327 95.....$$

The round-off error computer representation of the number π depends on how many digits are left out.

Number of digits	Approximation for π	Absolute error	Relative errror
1	3.100	0.141593	1.3239%
2	3.140	0.001593	0.0507%
3	3.142	0.000407	0.0130%

Truncation error

The notion of truncation error usually refers to errors introduced when a more complicated mathematical expression is "replaced" with a more elementary formula. This formula itself may only be approximates to the true values, and thus would not produce correct answers.

Truncation of an infinite series to a finite series to a finite number of terms leads to the truncation error. For example, the infinite Taylor series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!}$$

Check a few Taylor series approximations of the number e^x, for $x = 1$, $n = 2$, 3 and 4. Given that $e^1 = 2.718281$.

Order of n	Approximation for e^x	Absolute error	Relative error
2	2.500000	0.218281	8.030111%
3	2.666667	0.051614	1.898774%
4	2.708333	0.00995	0.365967%

There are still some errors not directly connected with the numerical methods themselves. These include blunders, formulation or model error, and data uncertainly.

📖 Revision: Significant figure

The significant figures of a number are those that can be used with confidence. They correspond to the number of certain digits plus one estimate digit.

<u>Example:</u>

How many significant figues in the following numbers?

a. 0.3 — 1 significant figure

b. 0.03 — 1 significant figure

c. 0.030 2 significant figures

d. 300 may be one, two or three significant figures, depending on whether the zeros are known with confidence.

e. 3×10^2 1 significant figure

f. 3.0×10^2 2 significant figures

g. 3.00×10^2 3 significant figures

The concept of significant figures will have relevance to the definition of accuracy and precision:

Precision refers to how closely individual measured or computed values agree with each other. Precision is governed by the number of digits being carried in the numerical calculations

Accuracy refers to how closely a number agrees with the true value of the number it is representing. Accuracy is governed by the errors in the numerical approximation.