CHAPTER TWO

PROBABILITY

2.0 Introduction

The concept of set and probability is very crucial to be known. Some events namely as mutually exclusive events, independent event, dependent even and also complementary event will be discussed in this chapter. Rules of multiplication and addition including Bayes' Theorem are also explained.

2.1 Probability of an event

For a given experiment, S denotes the sample space and A_1, A_2, … represent possible events. $P(A_i)$ is called the probability of A_i if the following properties are satisfied :

(i) $0 < P(A_i) < 1$,

(ii) $P(S) = 1$, *and*

(iii) $P(\phi) = 0$

If an experiment can result in any one of N different equally likely outcomes, and if exactly n of these outcomes correspond to event A, then the probability of an event A is

$$\boxed{\mathrm{P}(A) = \frac{n(A)}{n(S)} = \frac{n}{N}}$$

2.2 SOME PROPERTIES OF PROBABILITY

(i) If A is an event and A' is its complement, then

$$\boxed{\mathrm{P}(A') = 1 - \mathrm{P}(A)}$$

(ii) If A and B are any two events, then

$$\boxed{\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B).}$$

(iii) If A and B are mutually exclusive, then

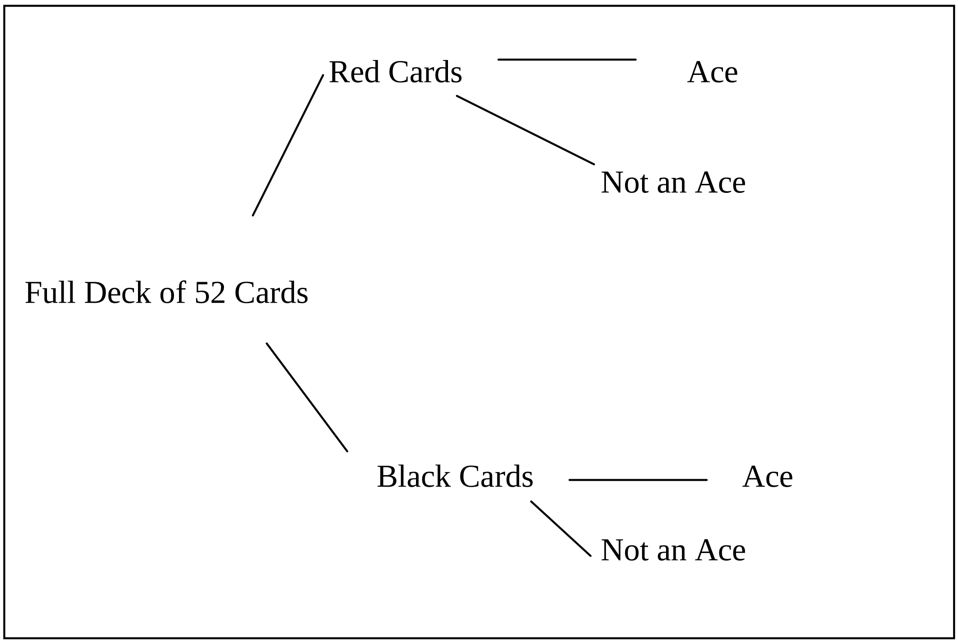
$$\boxed{P(A \cup B) = P(A) + P(B)}$$

2.3 VISUALIZING EVENTS

- **Contingency Tables**

	Ace	Not Ace	Total
Black	2	24	26
Red	2	24	26
Total	4	48	52

- **Tree Diagrams**



2.4 COMPUTING JOINT AND MARGINAL PROBABILITIES

Joint Probability :

- The probability of a joint event, A and B:

$$\boxed{P(A \text{ and } B) = \frac{\text{number of outcomes satisfying } A \text{ and } B}{\text{total number of elementary outcomes}}}$$

Marginal Probability :

- Computing a marginal (or simple) probability

$$\boxed{P(A) = P(A \text{ and } B_1) + P(A \text{ and } B_2) + \cdots + P(A \text{ and } B_k)}$$

Where B_1, B_2, …, B_k are k mutually exclusive and collectively exhaustive events

General Addition Rule :

$$\boxed{P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)}$$

If A and B are mutually exclusive, then P(A and B) = 0, so the rule can be simplified

$$\boxed{P(A \text{ or } B) = P(A) + P(B)}$$

For mutually exclusive events A and B

2.5 CONDITIONAL PROBABILITY

The probability of an event *B* occurring when it is known that some event *A* has occurred is called a conditional probability and its denoted by P*(B|A).*

Definition :

The conditional probability of *B* given *A*, denoted by P*(B|A)*, is defined by

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} \qquad \text{if} \quad P(A) \neq 0$$

2.6 MULTIPLICATION RULES

Multiplication rule for two events A and B:

$$\boxed{P(A \text{ and } B) = P(A|B)\,P(B)}$$

Note: If A and B are independent,then $P(A|B) = P(A)$ and the multiplication rule simplifies to $P(A \text{ and } B) = P(A)\,P(B)$

- **Independent Events :** Two events are independent if the occurrence of one ***does not affect*** the probability of the other occurring.

- **Dependent Events** : Two events are dependent if the first event affects the outcome or occurrence of the second event in a way the probability is changed

Definition :

Two events A and B are called ***independent events*** if $P(A \cap B) = P(A).P(B)$ OR

Two events A and B are called ***independent events*** if and only if $P(B|A) = P(B)$ and $P(A|B) = P(A)$

Otherwise, A and B are ***dependent***.

For any events A and B, the probability of the joint occurrence of A and B is given by

$$P(A \cap B) = P(A)\ P(B|A) = P(B)\ P(A/B)$$

This sometimes is referred to as the ***multiplication theorem of probability***.