CHAPTER THREE

DISCRETE RANDOM VARIABLES

3.0 Introduction

The concept of random variables is explained at the beginning. The functions of probability, cumulative, Binomial, hyper geometric and Poisson distributions are discussed. The expected value and variance are also discussed in detail.

3.1 Definition of random variables

A random variable can take of be assigned an integer value or whole number (by counting process).

3.2 The rules

The probability function p(x) should fulfil these rules:

i) For all values X, the probability value Pr(X = x) is fraction between 0 and 1 (inclusive)

ii) For all values of X, the total probabilities are equal to 1.

The mean and variance of a discrete probability distribution is given by:

$$\begin{aligned}\mu &= E(X)\\ &= x_1 \times \Pr(X = x_1) + \Pr(X = x_2) + ... + x_n \times \Pr(X = x_n)\\ &= \sum_{i=1}^{n} x_i \times \Pr(X = x_i)\end{aligned}$$

where,

$x_1, x_2, ..., x_n$ are all possible values of x which make probability distribution well defined, and $\Pr(X = x_1), \Pr(X = x_2), ..., \Pr(X = x_n)$ are the corresponding probabilities.

The variance and standard deviation of the distribution is given by one of the following formulas:

i) Variance : $\sigma^2 = \sum_{1}^{n} (x_i - \mu)^2 \times \Pr(X - x_i)$ or $\sigma^2 = \left(\sum_{1}^{n} x_i^{\,2} \times \Pr(X - x_i) \right) - \mu^2$

ii) Standard deviation : $\sigma = \sqrt{\sigma^2}$

3.3 Binomial Distribution

Binomial is a commonly used discrete probability distribution since many statiscal problems are deal with the situations referred to as repeated trial. The binomial experiment should have:

i) The experiment must have a fixed number of trials
ii) There are only two possible outcomes for each trial
iii) The trial must be independent
iv) The probability must remain constant for each trial.

Let S and F (success and failure) denote the two possible categories of all outcomes, then:

P(S) = p
P(F) = 1 – p = q

The probability distribution of the Binomial random variable X, representing the number of successes in n trials is given by the formula:

$$b(x;n,p) = f(x) = P(X = x) = \binom{n}{x} p^x q^{n-x} = \frac{n!}{(n-x)!} p^x q^{n-x} \qquad x = 0,1,2,\ldots,n$$

$$B(x,n,p) = F(x) = P(X \le x) = \sum_{k=0}^{x} b(k;n,p) \qquad x = 0,1,2,\ldots\ldots,n$$

Mean : $\mu = n.p$
Variance : $\sigma^2 = n.p.q$

x : number of success among n trials
p : probability of success in any one trial
q : probability of failure in any one trial.

$P(X = x) = f(x)$: probability of getting exact x success among the n trials.

3.4 The Hypergeometric Experiment

For hypergeometric distribution, we are interested in the probability of selecting x successes from the k items labelled success and $n - x$ failures from the $N - k$ items labelled failures when a random sample of size n is selected from N items. This hypergeometric experiment possesses the following two properties:

i) A random sample of size n is selected without replacement from N items.
ii) k of the N items may be classified as successes and $N - k$ are classified as failures.

The number X of successes of a hypergeometric experiment is called hypergeometric random variable.

3.5 Hypergeometric Probability Distribution

A set of N objects contains: k objects classified as successes and $N - k$ objects classified as failures. A sample of size n objects is selected randomly (without replacement) from the N objects, where $k \leq N$ and $n \leq N$.

Let the random variable X denote the number of successes in the sample. Then X is a hypergeometric random variable and

$$p(x) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}, \quad x = 0,1,2,....,n$$

Mean and Standard Deviation of the Hypergeometric Distribution

Mean : $\mu = np$

Variance : $\sigma^2 = np(1-q)\left(\frac{N-n}{N-1}\right)$ where $p = \frac{k}{N}$

3.6 Poisson Distribution

The Poisson distribution is a discrete probability distribution that applies to occurrences of some event over a specific interval. The random variable X is the distance, area, volume or some similar unit. Those experiment that possess the following properties are called Poisson experiment:

i) The occurrences must be random
ii) The occurrences must be independent form one interval to another
iii) The occurrences must be uniformly distributed over the interval being used.

The probability distribution of the Poisson random variable X, representing the number of outcomes occurring over an interval is given by the formula:

$$f(x;\lambda) = \frac{e^{-\lambda}\lambda^x}{x!} \qquad x = 0,1,2,..... \text{ and } \lambda > 0$$

Mean : $\mu = \lambda$

Variance : $\sigma^2 = \lambda$