# CHAPTER FOUR

# GEOMETRIC COORDINATES

## Introduction

In this chapter, we will discuss some methods for finding distances. These include the formula that used to find distances between the two points. The distances can also be calculated by using the ratio between two points. It will also apparent in this chapter that the distances between two parallel lines can be calculated using certain formula.

## Objectives

After completing these tutorials, students should be able to:

- Find the distance, the mid point and the slope of the given two points.
- Find the equation of the given straight line.
- Find the coordinates of R which divides the line in the given ratio.
- Find the perpendicular distance from a given point to a straight line.
- Find the perpendicular distance between two parallel lines.

**Question 1**

Given the points A (2,5) and B (-4,9). Find

(a) The distance between A and B.

**Solution:**

$$= \sqrt{\left(2-(-4)\right)^2 + (5-9)^2}$$

$$= \sqrt{\left(6\right)^2 + (-4)^2}$$

$$= 7.21$$

(b) The mid point of AB.

**Solution:**

$$\left(\frac{2+(-4)}{2}, \frac{5+9}{2}\right) = (-1, 7)$$

(c) The slope (gradient) of AB.

**Solution:**

$$= \frac{5-9}{2-(-4)} = \frac{-4}{6} = \frac{-2}{3}$$

(d) The equation of AB.

**Solution:**

$$y - 5 = \frac{-2}{3}(x-2)$$

$$3y - 15 = -2x + 4$$

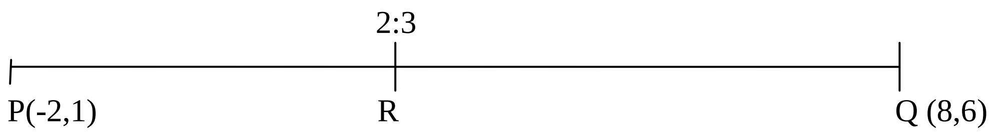
$$3y + 2x = 19$$

**Question 2**

Given the points P (-2,1) and Q (8.6). Find the coordinates of R which divides the line PQ.

(a) Internally in the ratio 2:3

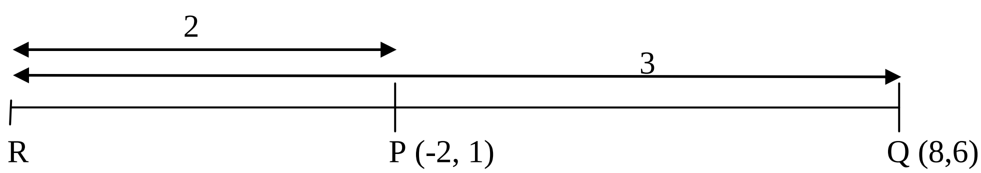
**Solution:**




$$R=\left(\frac{3(-2)+2(8)}{2+3},\frac{3(1)+2(6)}{2+3}\right)=\left(\frac{10}{5},\frac{15}{5}\right)=(2,3)$$

(b) Externally in the ratio 2:3

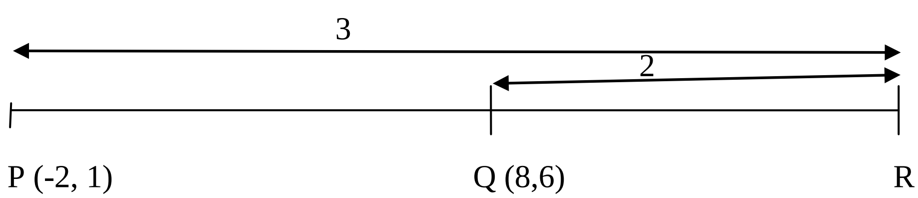
**Solution:**




$$R=\left(\frac{3(-2)-2(8)}{3-2},\frac{3(1)-2(6)}{3-2}\right)=\left(\frac{-22}{1},\frac{-9}{1}\right)=(-22,-9)$$

(c) Externally in the ratio 3:2

**Solution:**




$$R=\left(\frac{2(-2)-3(8)}{2-3},\frac{2(1)-3(6)}{2-3}\right)=\left(\frac{-28}{-1},\frac{-16}{-1}\right)=(28,16)$$

**Question 3**

Find the equation of a straight line which passes through the point (-3,1) and is perpendicular to the straight line x + 2y - 4 = 0.

**Solution:**

$$x + 2y - 4 = 0$$

$$2y = -x + 4$$

$$y = -\frac{x}{2} + 2$$

The gradient is $-\frac{1}{2}$

Let m be the gradient of the perpendicular line.

$$m_1(-\frac{1}{2}) = -1$$

$$m_1 = 2$$

Hence, the equation of the straight line passing through the point (-3, 1) with gradient 2 is:

$$y - 1 = 2(x - (-3))$$

$$y - 1 = 2x + 6$$

$$y = 2x + 7$$

**Question 4**

Find the equation of a straight line which :

(a) passes through the point A (4,-1) and B (0,1).

**Solution:**

$$m = \frac{1-(-1)}{0-4} = -\frac{1}{2}$$

Hence the equation of the line passing through A and B is

$$y - 1 = -\frac{1}{2}(x - 0)$$

$$y = -\frac{1}{2}x + 1$$

$$2y + x = 1$$

(b) passes through the point P (4,-1) and has gradient 2. Show that these two straight lines are perpendicular to one another.

**Solution:**

$$y-5=2(x-1)$$
$$y=2x+3$$

Product of the gradients of the two lines is $2\times(-\frac{1}{2})=-1$, hence the two lines are perpendicular to one another.

**Question 5**

Find the perpendicular distance from point A (2,3) to straight line 3x + 4y = 1.

**Solution:**

By usibg the formula $d=\frac{|ah+bk+c|}{\sqrt{a^2+b^2}}$ for $3x+4y-1=0$, we have

$a=3,\ b=4,\ c=-1,\ h=2,\ k=3$

Hence, $d=\frac{|3(2)+4(3)+(-1)|}{\sqrt{3^2+4^2}}\quad=\frac{|17|}{5}\quad=\frac{17}{5}\quad=3\tfrac{2}{5}$

**Question 6**

The coordinates of the point P, Q, R are given by (-2,1), (1,4) and 4k respectively. Find the values of k, if

(a) P, Q and R lie on a line

**Solution:**

Gradient PQ = gradient QR

$$m_{PQ}=\tfrac{4-1}{1-(-2)}\quad=1$$
$$m_{PQ}=\tfrac{4-1}{1-(-2)}\quad=1$$
$$m_{QR}=\frac{k-4}{4-1}=\frac{k-4}{3}$$
$$Thus,\ \frac{k-4}{3}=1\qquad\therefore k=7$$

(b) PQ = QR

**Solution:**

$$\sqrt{(1-(-2))^2+(4-1)^2} = \sqrt{(4-1)^2+(k-4)}$$

$$\sqrt{9+9} = \sqrt{9+k^2-8k+16}$$

$$18 = k^2-8k+25$$

$$Thus, \quad k^2-8k+7=0$$

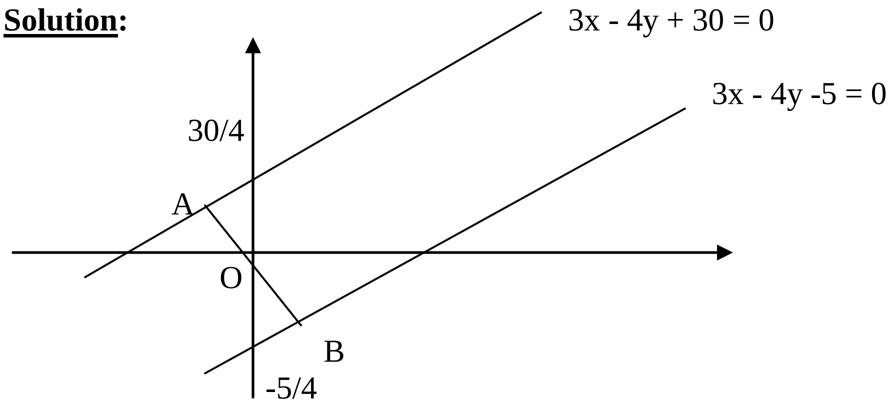
$$(k-7)(k-1)=0$$

$$k=1,\ 7$$

**Question 7**

Find the perpendicular distance between the parallel lines 3x – 4y – 5 = 0 and 3x – 4y + 30 = 0

**Solution:**




$$OA = \frac{|3(0)-4(0)+30|}{\sqrt{3^2+4^2}} \quad = \frac{30}{5} \quad = 6$$

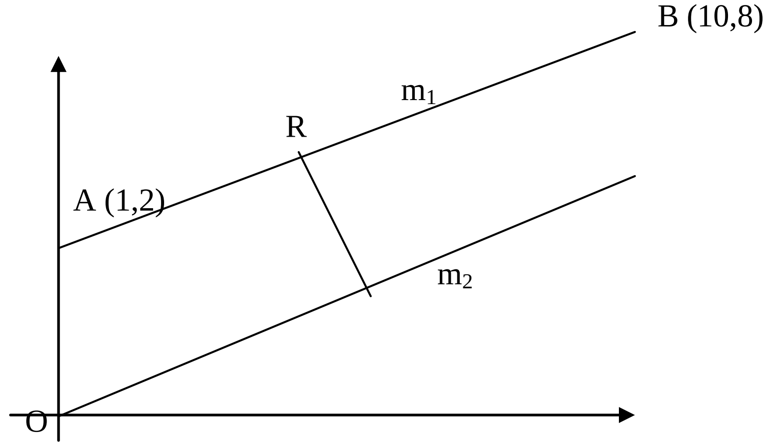
$$OB = \frac{|3(0)-4(0)-5|}{\sqrt{3^2+4^2}} \quad = \frac{5}{5} \quad = 1$$

The perpendicular distance between the parallel lines = AB = OA + OB = 7

**Question 8**

The point R divides internally the line joining the points A (1, 2) and B (10, 8) in the ratio 1 : 4. Find the cooordinates of the point R and the perpendicular distance of R to the line passing through the origin O and parallel to AB.

**Solution:**




$$R = \left( \frac{nx_1 + mx_2}{n+m}, \frac{ny_1 + my_2}{n+m} \right) = \left( \frac{4(1)+1(10)}{5}, \frac{4(2)+1(8)}{5} \right) = \left( \frac{14}{5}, \frac{16}{5} \right)$$

Gradient of AB = $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{8-2}{10-1} = \dfrac{6}{9} = \dfrac{2}{3}$

Equation of the line passing through the origin and parallel to line AB is:

$m_1 = m_2 = \dfrac{2}{3}$

$$y - 0 = \frac{2}{3}(x - 0)$$
$$y = \frac{2}{3}x$$
$$3y - 2x = 0$$

The perpendicular distance of R to the line passing through the origin O and parallel to AB is:

$$= \frac{|3y - 2x|}{\sqrt{3^2 + 2^2}} \quad \text{where } x = 14/5,\ y = 16/5$$

$$= \frac{\left|3\left(\frac{16}{5}\right) - 2\left(\frac{14}{5}\right)\right|}{\sqrt{3^2 + 2^2}} = \frac{4}{\sqrt{13}} = \frac{4\sqrt{13}}{13}$$