CHAPTER 5 APPLICATIONS OF INTEGRATION

5.1 Geometrical Interpretation-Definite Integral (page 362)

5.2 Area of a Region (page 369)

5.2.1 Area of a Region Under a Graph (page 369)

Figure 5.7 shows the region bounded by the curve $y = f(x)$, the x-axis, and the lines $x = a$ and $x = b$. This region is located above the x-axis. The area of this region is given by

$$\int_a^b f(x)\,dx$$

Figure 5.8 shows the region bounded by the curve $y = g(x)$, the x-axis, and the lines $x = a$ and $x = b$. This region is located below the x-axis. The definite integral $\int_a^b f(x)\,dx$ has a negative value. Since the area is always a positive quantity, the area of this region is written as

$$\left| \int_a^b g(x)\,dx \right|$$

Figure 5.9 shows the region bounded by the curve $x = u(y)$, the y-axis, and the lines $y = c$ and $y = d$. This region is located on the right hand side of the y-axis. The area of this region is given by

$$\int_c^d u(y)\,dy$$

Example 5.3 (page 370):

Find the area of the region bounded by the curve $y = 2 - x^2$, the x-axis and the lines $x = 0$ and $x = 1$.

Example 5.4 (page 371):

Find the area bounded by the lines $y = 2 - x$, $x = 3$, $x = 4$ and x-axis.

Example 5.8 (page 374):

Find the area of the region in the first quadrant bounded by the curve $y = \frac{2}{x}$, y-axis with lines $y = 2$ and $y = 4$.

5.2.2 Area of the Region between Two Curves (page 374)

(page 375)

Given two curves $y = f(x)$ and $y = g(x)$, let $g(x) \le f(x)$ for [a, b], the area of the region that lies between these two curves in the interval [a, b] (which is located above the x-axis) is

$$\int_a^b f(x)\,dx - \int_a^b g(x)\,dx = \int_a^b \left[f(x) - g(x)\right] dx$$

The above formula is also valid for the graph that is located below the x-axis.

Definition 5.2 (Area Between Two Curves) (page 376)

If $f(x)$ and $g(x)$ are continuous in the interval [a, b] and $g(x) \le f(x)$ for all x in [a, b], then the area of the region bounded by the curves $y = f(x)$ and $y = g(x)$ between the lines $x = a$ and $x = b$ is given by

$$A = \int_a^b \{f(x) - g(x)\}\, dx.$$

Example 5.10 (page 376):

Find the area of the region bounded by the curve $y = x^2 + 2$ and the lines $y = -x$, $x = 0$ and $x = 1$.

Example 5.15 (page 380):

Find the area of the region bounded by the curves $y^2 = 3 - x$ and the line $y = x - 1$.

Definition 5.3 (Area Between Two Curves about y-Axis) (page 383)

If $u(y)$ and $v(y)$ are continuous in the interval [c, d] and $v(y) \le u(y)$ for all y in [c, d], then the area of the region bounded by the curves $x = u(y)$ and $x = v(y)$ between the lines $y = c$ and $y = d$ is given by

$$A = \int_c^d \{u(y) - v(y)\}\, dy.$$

Example 5.17 (page 383):

Find the area of the region bounded by the curves $y^2 = 3 - x$ and a line $y = x - 1$.

5.3 Volume of Revolution (page 387)

If a plane region is revolves about a line then a solid object is generated which is called the **solid of revolution**, and the line is called the **axis of revolution**.

Definition 5.4 (Volume of Revolution about x-Axis) (page 388)

Let $f(x)$ be a non-negative and continuous function in the interval [a, b]. If the region between this curve, the x-axis and the lines $x = a$ and $x = b$ revolves 360° about the x-axis, then the volume of the solid generated is

$$V = \pi \int_a^b [f(x)]^2 \, dx.$$

Example 5.20 (page 388):

Find the volume of the solid of revolution when the region bounded by the parabola $y = 2\sqrt{x}$ and x-axis within the interval [0, 4] revolves 360° about the x-axis.

Definition 5.5 (Volume of Revolution about y-Axis) (page 390)

Let $u(y)$ be a non-negative and continuous function in the interval [c, d]. If the region bounded by $x = u(y)$, the y-axis and the lines $y = c$ and $y = d$ revolves 360° about the y-axis, the volume of the solid generated is

$$V = \pi \int_c^d [u(y)]^2 \, dy .$$

Example 5.23 (page 390):

Find the volume of the solid of revolution when the region bounded by the curve $y = x^2 + 1$, the lines $y = 1$, $y = 2$ and the y-axis revolves 360° about the y-axis.

Definition 5.6 (Volume of Revolution about x-Axis between two Curves) (page 391)

Let $f(x)$ and $g(x)$ be non-negative and continuous functions in the interval [a, b] and $g(x) \le f(x)$ for all x in the interval [a, b]. The volume of revolution when the region bounded by $y = f(x)$, $g(x)$, $x = a$ and $x = b$, revolves 360° about the x-axis is

$$V = \pi \int_a^b \left\{ [f(x)]^2 - [g(x)]^2 \right\} dx.$$

Example 5.24 (page 391):

Find the volume of the solid of revolution when the region bounded by the curve $y^2 = 8x$ and $y = x^2$ revolves at 360° about the x-axis.

Definition 5.7 (Volume of Revolution about y-Axis Between Two Curves) (page 392)

Let $u(y)$ and $v(y)$ be a non-negative and continuous function in the interval [c, d] and $v(y) \leq u(y)$ for all y in the interval [c, d]. The volume of the solid generated when the region bounded by the curves $x = u(y)$, $x = v(y)$, $y = c$ and $y = d$ revolves 360° about the y-axis is

$$V = \pi \int_c^d \left\{ [u(y)]^2 - [v(y)]^2 \right\} dy.$$

Example 5.25 (page 392):

Find the volume of the solid of revolution when the region bounded by the curve $y^2 = 8x$ and $y = x^2$ revolves at 360° about the x-axis.

Exercise at home: (Tutorial 10)

(Page 385) Quiz 5B: no. 1, 3, 4, 7.

(Page 394) Quiz 5C: no. 1(a), 1(c), 2(b), 2(e).

(Page 395) Exercise 5: no. 23, 33, 43.