CHAPTER FIVE

SAMPLING DISTRIBUTIONS

5.0 Introduction

This chapter focused on the population distribution. Sampling distribution of sample mean, Sampling distribution of sample proportion and the use of the central limit theorem in facilitating advanced inferential analysis.

5.1 Basic Definitions:

A ***population*** is a collection or set, of all individuals, objects, or items of interest.

Population Parameters :

Mean: μ,
Variance: σ^2 ,
Proportion, p

A ***sample*** is a portion, or part, of the population of interest.

A ***statistic*** is any quantity whose value can be calculated from a sample data.

Sample Statistics :

Sample mean, $\bar{x}$,
Sample variance, s^2,
Sample proportion, $\hat{p}$

The probability distribution of a sample statistic is called a ***sampling distribution***.

5.2 Sample Distribution for Sample Mean

If $X_1, X_2, ..., X_n$ constitute a random sample of size n, then the ***sample mean*** is defined by the statistic

$$\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

If $X_1, X_2, ..., X_n$ be a random sample of size *n* from a **Normal** population with mean μ and variance σ^2, then

$$E(\overline{X}) = \mu_{\overline{X}} = \mu$$

$$Var(\overline{X}) = \sigma_{\overline{X}}^2 = \frac{\sigma^2}{n}$$

Note:

- **Note :** $X_i \sim N(\mu, \ \sigma^2)$ and $\overline{X} \sim N(\mu, \ \frac{\sigma^2}{n})$

If we are sampling from a population with **unknown** distribution, either finite or infinite, the sampling distribution of $\overline{X}$ will still be approximately normal with mean μ and variance $\frac{\sigma^2}{n}$, ***provided*** that the sample size is ***large***, $n \geq 30$.

5.3 Central Limit Theorem

Let $X_1, X_2, ..., X_n$ be a random sample of size *n* from a distribution with mean μ and variance σ^2. Then if *n* is sufficiently large ($n \geq 30$), $\overline{X}$ has approximately a normal distribution with $\mu_{\overline{X}} = \mu$ and $\sigma_{\overline{X}}^2 = \frac{\sigma^2}{n}$. i.e.

$$\overline{X} \sim N(\mu, \ \frac{\sigma^2}{n})$$

then the limiting distribution of

$$Z = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}}$$

as $n \to \infty$, is the standard normal distribution, $Z \sim N(0,1)$ or $\overline{X}$ approximately follows a normal distribution with mean μ and standard deviation $\sigma / \sqrt{n}$.

5.4 Sampling Distribution Of The Sample Proportion

In practice, the proportion statistic is calculated as follow:

If $\hat{p} = \dfrac{X}{n}$ be the sample proportion of successes with

$$\boxed{\text{mean } \mu_{\hat{p}} = p}$$

and

$$\boxed{\text{standard deviation : } \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}\,.}$$

Then statistic for sample proportion

$$\boxed{Z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} = \frac{\hat{p} - p}{\sqrt{\dfrac{p(1-p)}{n}}}\,,}$$

with limiting distribution $Z \sim N(0,1)$ as $n \to \infty$.