CHAPTER SIX

ESTIMATION

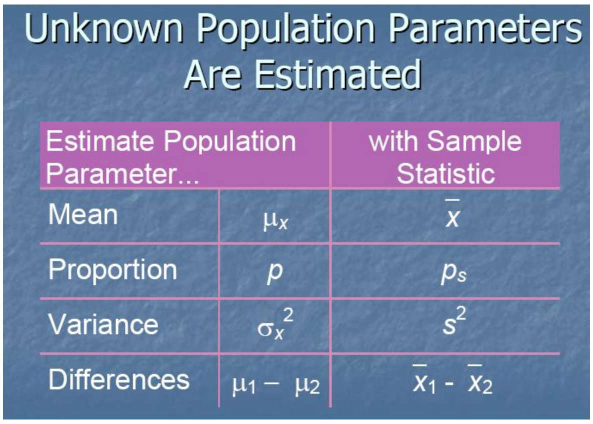
6.0 Introduction

The concept of estimation is explained in the beginning. The estimation of point and interval will be discussed in detail. Sample size can be determined by considering the standard deviation and the confidence level.

6.1 Basics of Point Estimation

A point estimator consists of a single sample statistic that estimates an unknown population parameter.

For instance, the sample mean $\bar{x}$ is a ***point estimator*** of population mean μ, the sample variance s^2 is a ***point estimator*** of population variance σ^2, $\hat{p}$ is a ***point estimator*** of p.


Unknown Population Parameters Are Estimated

Estimate Population Parameter...		with Sample Statistic
Mean	μ_x	$\bar{x}$
Proportion	p	p_s
Variance	σ_x^2	s^2
Differences	$\mu_1 - \mu_2$	$\bar{x}_1 - \bar{x}_2$

6.2 Single Sample : Estimating the Mean

Case 1 : *"σ is known" or "σ is unknown but* $n \geq 30$ *"*

If $\bar{x}$ is the value of the mean of a random sample of size n from a normal population with known variance σ^2, then

$$\bar{x} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \;<\; \mu \;<\; \bar{x} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

is a $(1-\alpha)100\%$ confidence interval for μ

***** Replace σ^2 with s^2 if σ^2 is unknown.***

Case 2 : *“σ is unknown and $n \le 30$ ”*

If $\bar{x}$ and s are the mean and standard deviation of a random sample of size n from a normal population with unknown variance σ^2, then

$$\bar{x} - t_{\alpha/2,\; \nu=n-1}\frac{s}{\sqrt{n}} \;<\; \mu \;<\; \bar{x} + t_{\alpha/2,\; \nu=n-1}\frac{s}{\sqrt{n}}$$

is a $(1-\alpha)100\%$ confidence interval for μ

<u>Theorem :</u>

If $\bar{x}$ is used as an estimate of μ, we can then be $(1-\alpha)100\%$ confident that the error will not exceed $Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$.

<u>Theorem :</u>

If $\bar{x}$ is used as an estimate of μ, we can then be $(1-\alpha)100\%$ confident that the error will not exceed a specified amount e when the sample size is

$$n = \left(\frac{Z_{\alpha/2}\sigma}{e}\right)^2$$

6.3 Single sample : Estimating a proportion

A point estimator of the population p in a binomial experiment is given by the statistic $\hat{p} = \frac{X}{n}$ where X represent the number of successes in n trials.

If $n\hat{p} \geq 5$ and $n\hat{q} \geq 5$, by central limit theorem, for *n* sufficiently large, $\hat{p}$ is approximately normally distributed with mean $\mu_{\hat{p}} = E(\hat{p}) = E\left(\frac{X}{n}\right) = \frac{np}{n} = p$ and variance $\sigma^2_{\hat{p}} = \text{var}\left(\frac{X}{n}\right) = \frac{1}{n^2}Var(X) = \frac{npq}{n^2} = \frac{pq}{n}$.

Therefore,

$$Z = \frac{\hat{p} - p}{\sqrt{pq/n}} \sim N(0,\ 1)$$

If $\hat{p}$ is the proportion of successes in a random sample of size *n*, then an approximate $(1-\alpha)100\%$ confidence interval for the binomial parameter *p* is given by

$$\hat{p} - Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} \quad < \quad p \quad < \quad \hat{p} + Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

Theorem :

If $\hat{p}$ is used as an estimate of *p*, we can then be $(1-\alpha)100\%$ confident that the error will not exceed $Z_{\alpha/2}\sqrt{\frac{pq}{n}}$.

Theorem :

If $\hat{p}$ is used as an estimate of *p*, we can then be $(1-\alpha)100\%$ confident that the error will not exceed a specified amount *e* when the sample size is

$$n = \left(\frac{Z_{\alpha/2}}{e}\right)^2 pq$$