CHAPTER SIX

TRIGONOMETRY

Introduction

In this chapter, we will learn how to find angle measurement in degree and radian. Introduce trigonometric functions using coordinate systems and their graphs. Furthermore we will learn to solve trigonometric equation using fundamental trigonometric identities. Then in this topic we will learn how to solve the right angled triangles by two sides, by a side and an acute angle and more difficult applications. We will also discuss about basic relations between elements of triangle which contain the law of sine, the law of cosine and the area of a triangle.

Objectives

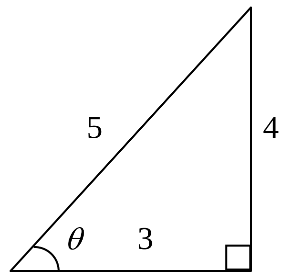
After completing these tutorials, students should be able to:

- Find the values of the six trigonometric ratios of the angle θ in the given triangle.
- Evaluate the value for $\sin\theta$, $\cos\theta$ and $\tan\theta$ for the given angle.
- Verify the given trigonometry identities.
- Use sine and cosine rules to solve triangle problems

Question 1

Find the values of the six trigonometric ratios of the angle θ in the triangle shown.

(a)



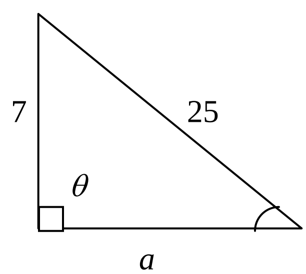
Solution:

$\sin\theta = \frac{4}{5}$; $\quad \cos ec\theta = \frac{1}{\sin\theta} = \frac{5}{4}$

$\cos\theta = \frac{3}{5}$; $\quad \sec\theta = \frac{1}{\cos\theta} = \frac{5}{3}$

$\tan\theta = \frac{4}{5}$; $\quad \cot\theta = \frac{1}{\tan\theta} = \frac{3}{4}$

(b)



Solution:

Using pythagores theorem, $a = \sqrt{25^2 - 7^2} = \sqrt{576} = 24$

$\sin\theta = \frac{7}{25}$; $\cos ec\theta = \frac{1}{\sin\theta} = \frac{25}{7}$

$\cos\theta = \frac{24}{25}$; $\sec\theta = \frac{1}{\cos\theta} = \frac{25}{24}$

$\tan\theta = \frac{7}{24}$; $\cot\theta = \frac{1}{\tan\theta} = \frac{24}{7}$

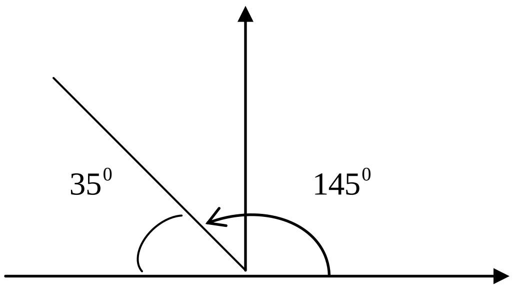
Question 2

Evaluate for $\sin\theta, \cos\theta, \tan\theta$ for each of the following angle

(a) 145^0

Solution:

145^0 is in quadrant II, base angle is 35^0



$\sin 145^0 = \sin 35^0 = 0.5736$

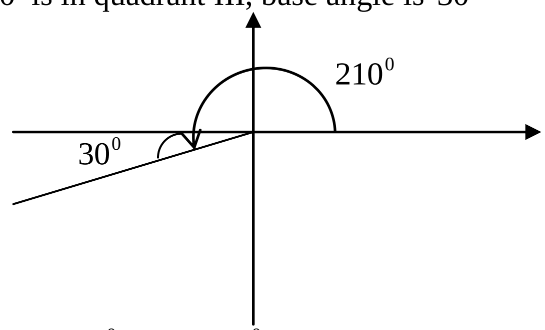
$\cos 145^0 = -\cos 35^0 = -0.8192$

$\tan 145^0 = -\tan 35^0 = -0.7002$

(b) 210^0

Solution:

210^0 is in quadrant III, base angle is 30^0



$\sin 210^0 = -\sin 30^0 = -0.5$

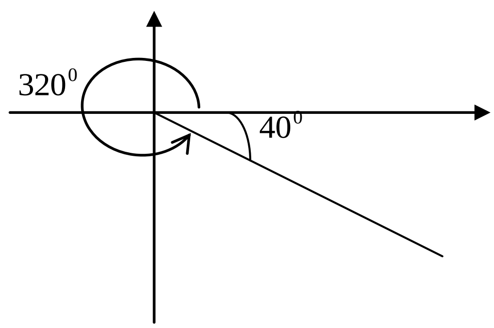
$\cos 210^0 = -\cos 30^0 = -0.8660$

$\tan 210^0 = \tan 30^0 = 0.5774$

(c) 320^0

Solution:

320^0 is in quadrant IV, base angle is 40^0



$$\sin 320^0 = -\sin 40^0 = -0.6428$$
$$\cos 320^0 = \cos 40^0 = 0.7660$$
$$\tan 320^0 = -\tan 40^0 = -0.8391$$

Question 3

Evaluate

(a) $\sin(-45^0)$

Solution:

$$\sin(-45^0) = -\sin 45^0 = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

(b) $\cos(-45^0)$

Solution:

$$\cos(-45^0) = \cos 45^0 = \frac{\sqrt{2}}{2}$$

(c) $\tan(-45^0)$

Solution:

$$\tan(-45^0) = -\tan 45^0 = -1$$

Question 4

Verify the following identities

(a) $\sin\theta\cot\theta = \cos\theta$

Solution:

$$\begin{aligned}
&\sin\theta\cot\theta \\
&= \sin\theta\cdot\frac{1}{\tan\theta} \\
&= \sin\theta\cdot\frac{1}{\frac{\sin\theta}{\cos\theta}} \\
&= \sin\theta\cdot\frac{\cos\theta}{son\theta} \\
&= \cos\theta
\end{aligned}$$

(b) $\dfrac{\tan x}{\cos ecx} = \sec x - \cos x$

Solution:

$$\begin{aligned}
&\frac{\tan x}{\cos ecx} \\
&= \frac{\tan x}{\frac{1}{\sin x}} \\
&= \tan x \sin x \\
&= \frac{\sin x}{\cos x}\cdot \sin x \\
&= \frac{1-\cos^2 x}{\cos x} \\
&= \frac{1}{\cos x} - \cos x \\
&= \sec x - \cos x
\end{aligned}$$

(c) $\frac{\cos x}{\sec x}+\frac{\sin x}{\cos ecx}=1$

Solution:

$$\frac{\cos x}{\sec x}+\frac{\sin x}{\cos ecx}$$
$$=\frac{\cos x}{\frac{1}{\cos x}}+\frac{\sin x}{\frac{1}{\sin x}}$$
$$=\cos^2 x+\sin^2 x$$
$$=1$$

(d) $(1-\cos\beta)(1+\cos\beta)=\frac{1}{\cos ec^2\beta}$

Solution:

$$(1-\cos\beta)(1+\cos\beta)$$
$$=1^2-\cos^2\beta$$
$$=1-\cos^2\beta$$
$$=\sin^2\beta$$
$$=\frac{1}{\cos ec^2\beta}$$

Question 5

Verify the following identities

(a) $\frac{1-\cos\alpha}{\sin\alpha}=\frac{\sin\alpha}{1+\cos\alpha}$

Solution:

$$\frac{1-\cos\alpha}{\sin\alpha}$$
$$=\frac{1-\cos\alpha}{\sin\alpha}\cdot\frac{1+\cos\alpha}{1+\cos\alpha}$$
$$=\frac{1-\cos^2\alpha}{\sin\alpha(1+\cos\alpha)}$$
$$=\frac{\sin^2\alpha}{\sin\alpha(1+\cos\alpha)}$$
$$=\frac{\sin\alpha}{1+\cos\alpha}$$

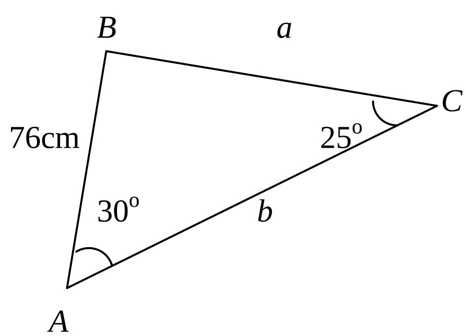
(b) $\dfrac{(\sin x + \cos x)^2}{\sin x \cos x} = 2 + \sec x \cos ecx$

Solution:

$$\frac{(\sin x + \cos x)^2}{\sin x \cos x}$$
$$= \frac{\sin^2 x + \cos^2 x + 2\sin x \cos x}{\sin x \cos x}$$
$$= \frac{1 + 2\sin x \cos x}{\sin x \cos x}$$
$$= \frac{1}{\sin x \cos x} + \frac{2\sin x \cos x}{\sin x \cos x}$$
$$= 2 + \sec x \cos ecx$$

Question 6

Solve the triangle in figure below:



Solution:

$B = 180^\circ - (30^\circ + 25^\circ) = 125^\circ$

Since side c is known to find side a we use the Sine Rule

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 30^\circ}{a} = \frac{\sin 25^\circ}{c}$$
$$\therefore a = \frac{76 \times \sin 30^\circ}{\sin 25^\circ} = 89.92cm$$

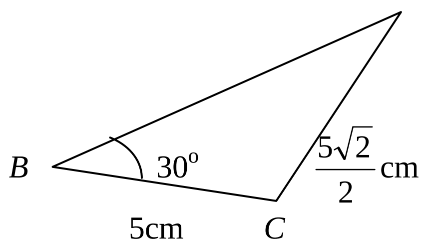
Similarly to find *b*, we use

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$b = c\frac{\sin B}{\sin C} = \frac{76 \times \sin 125^\circ}{\sin 25^\circ} = 147.31cm$$

Question 7

Solve the triangle *ABC* where $\angle B = 30^\circ, b = \frac{5\sqrt{2}}{2}cm$ and a = 5cm.



Solution:

$$\frac{\sin A}{5} = \frac{\sin 30^\circ}{\frac{5\sqrt{2}}{2}}$$

$$\sin A = \frac{5 \times 2}{\frac{5\sqrt{2}}{2}}$$

$$\sin A = \frac{1}{\sqrt{2}}$$

$$A = 45^\circ$$

$$\therefore \angle C = 180^\circ - 45^\circ - 30^\circ = 105^\circ$$

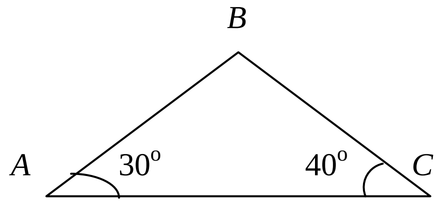
$$\frac{c}{\sin 105^\circ} = \frac{5}{\sin 45^\circ}$$

$$c = \frac{5 \sin 45^\circ}{\sin 45^\circ} = \frac{5 \sin 75^\circ}{\sin 45^\circ} = 6.830cm$$

Question 8

Solve the following triangles. Find also their areas.

(a)



Solution:

$\angle B = 180^\circ - (30^\circ + 40^\circ) = 110^\circ$

To find the side *AB*, use the Sine Rule

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

$$c = \frac{b \sin C}{\sin B} = \frac{60 \sin 40^\circ}{\sin 110^\circ} = 41.0424cm$$

For the side *BC*

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$a = \frac{60 \sin 30^\circ}{\sin 110^\circ} = 31.93cm$$

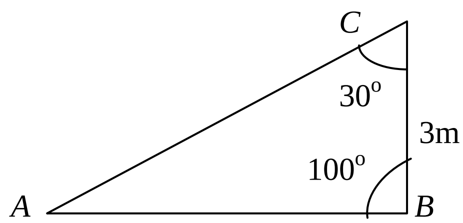
Areas of ΔABC

$$= \frac{1}{2} bc \sin A$$

$$= \frac{1}{2} \times 60 \times 41.0424 \times \sin 30^\circ$$

$$= 615.6cm^2$$

(b)



Solution:

$\angle A = 180^\circ - (30^\circ + 100^\circ) = 50^\circ$

To find the side *AC* use the Sine Rule

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$b = \frac{a \sin B}{\sin A} = \frac{3 \sin 30^\circ}{\sin 50^\circ} = 1.9581m$$

For the side *AB*

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$c = \frac{a \sin C}{\sin A} = \frac{3 \sin 100^\circ}{\sin 50^\circ} = 3.8567m$$

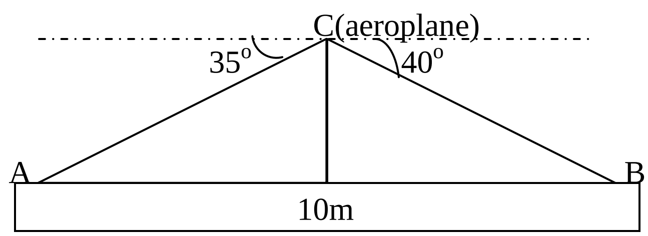
Area of ΔABC

$$= \frac{1}{2} ab \sin AC$$

$$= \frac{1}{2} \times 3 \times 1.9581 \times \sin 100^\circ = 2.893m^2$$

Question 9

A pilot is flying over a straight highway. He determines the angles of depression to two mileposts, 10m apart, to be 35^o and 40^o as shown in the figure.



(a) Find the distance of the plane from point *A*

Solution:

$\angle ACB = 180^\circ - (35^\circ + 40^\circ) = 105^\circ$

$\angle CAB = 35^\circ, \angle CBA = 40^\circ$

$$\frac{AC}{\sin 40^\circ} = \frac{10}{\sin 105^\circ}$$

$$AC = \frac{10\sin 40^\circ}{\sin 105^\circ} = 6.6546m$$

(b) Find the elevation of the plane

Solution:

Let the elevation be *t* meter

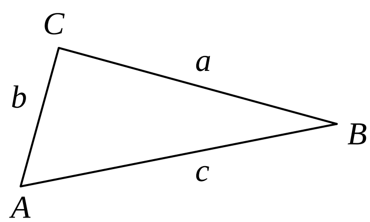
We have,

$$\sin 35^\circ = \frac{t}{6.5646}$$

$$t = 6.5646\sin 35^\circ = 3.8169m$$

Question 10

The sides of triangle are $a = 5$, $b = 8$ and $c = 12$. Find the angles of the triangle.



Solution:

First find $\angle A$. From the Cosine Rule we have $a^2 = b^2 + c^2 - 2bc\cos A$.
Solving for cos *A*, we get

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{8^2 + 12^2 - 5^2}{2(8)(12)} = \frac{183}{192} = 0.9531$$

We find that $\angle A = 17.62^\circ$

In the same way, we get

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{5^2 + 12^2 - 8^2}{2(5)(12)} = 0.875$$

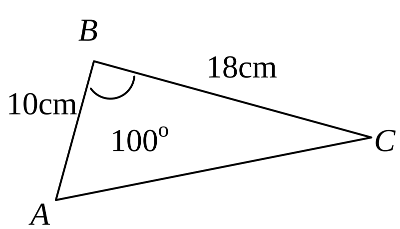
$$\therefore B = \cos^{-1}(0.875) = 28.96^\circ$$

$$\cos c = \frac{a^2 + b^2 - c^2}{2ab} = \frac{5^2 + 8^2 - 12^2}{2(5)(8)} = -0.6875$$

$$\therefore C = \cos^{-1}(-0.6875) = 133.43^\circ$$

Question 11

Solve triangle *ABC*



Solution:

Using the Cosine Rule

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$= 18^2 + 10^2 - 2(10)(18)\cos 100^\circ$$

$$= 324 + 100 - 360(-0.1736)$$

$$= 486.496$$

$$\therefore b = \sqrt{486.496} = 22.05cm$$

Using the Sine Rule, we have

$$\frac{\sin A}{18} = \frac{\sin B}{22.05}$$

$$\sin A = \frac{18 \sin 100^\circ}{22.05} = 0.8039$$

$$\therefore \angle A = 53.5^\circ$$

We get $\angle C = 180^\circ - (100^\circ + 53.5^\circ) = 26.5^\circ$