CHAPTER SEVEN

HYPOTHESIS TESTING

7.0 Introduction

This chapter discusses the hypothesis testing of the population mean parameter. The discussions include the case where the population variance is unknown. Some examples of the application of hypothesis testing on population proportion will also be explained.

7.1 Hypothesis

Definition :

Null hypothesis is a hypothesis which is tested for possible rejection under the assumption that it is true and is denoted by H_0

Alternative hypothesis is a complimentary hypothesis to null hypothesis and is denoted by H_1

7.2 Procedures of conducting the tests of hypotheses

Step 1 Formulate H_0 : $\theta \le \theta_0$, $\theta \ge \theta_0$, $\theta = \theta_0$

Step 2 Formulate H_1 : $\theta > \theta_0$, $\theta < \theta_0$, $\theta \ne \theta_0$

Step 3 Specify α

Step 4 Determine a critical region of size α. (If the conclusion is to be based on a *P*-value, it is not necessary to state the critical region)

Step 5 Compute the value of the test statistic from the sample data

Step 6 Conclusions : Reject H_0 if the statistic has a value in the critical region (or if the computed *P*-value is less than or equal to the desired significance level α); otherwise, do not reject H_0.

- For 2 tailed test, *P*-value $= 2P\left(Z > Z_{\alpha/2} \right) = 2P\left(Z < -Z_{\alpha/2} \right)$
- For 1 tailed test, *P*-value $= P(Z > Z_\alpha) = P(Z < -Z_\alpha)$
- **Note :** If *P*-value $\le \alpha$, $\Rightarrow$ reject H_0

7.3 Single Sample : Test concerning the Mean

Case 1 : ***"σ is known" or "σ is unknown but $n \geq 30$"***

Suppose we have a sample of size *n* taken from a population whose mean is μ and variance σ^2. We want to test whether this sample is taken from a population whose mean is μ_0. We know that the sample mean $\overline{X} \sim N\left(\mu,\ \dfrac{\sigma^2}{n}\right)$ if *n* is large.

For ***two-tailed*** hypothesis,

(i) $H_0 :\ \mu = \mu_0$

(ii) $H_1 :\ \mu \neq \mu_0$

(iii) $\alpha = 0.05$

(iv) Critical region : Reject H_0 if $Z > Z_{\alpha/2}$ *or* $Z < -Z_{\alpha/2}$

(v) Test statistic $Z = \dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}}$

- **Note :** ***Replace σ with s if σ is unknown***

Case 2 : ***"σ^2 is unknown and $n < 30$"***

Suppose we have a sample of size *n* taken from a normal population whose mean is μ and variance unknown. We want to test whether this sample is taken from a population whose mean is μ_0.

For two-tailed hypothesis,

(i) $H_0 :\ \mu = \mu_0$

(ii) $H_1 :\ \mu \neq \mu_0$

(iii) $\alpha = 0.05$

(iv) Critical region : Reject H_0 if $T > t_{\alpha/2,\ n-1}$ *or* $T < -t_{\alpha/2,\ n-1}$

(v) Test statistic $T = \dfrac{\overline{X} - \mu}{s/\sqrt{n}}$

7.4 One Sample : Test on a Single Proportion

To test whether a random sample of size n, where n is large, with the proportion of "successes" $\hat{p}$, could be drawn from a population with the proportion of "successes" p.

For two-tailed hypothesis,

(i) $H_0: \quad p = p_0$

(ii) $H_1: \quad p \neq p_0$

(iii) $\alpha = 0.05$

(iv) Critical region : Reject H_0 if $Z > Z_{\alpha/2}$ *or* $Z < -Z_{\alpha/2}$

Test statistic $$Z = \frac{\hat{p} - p}{\sqrt{pq/n}}$$