CHAPTER EIGHT

ESTIMATION AND HYPOTHESIS TESTING FOR TWO POPULATIONS

8.0 Introduction

A discussion on the inferences about the difference between two population means for large and small independent samples for equal and unequal standard deviations. Beside that, the inference about the difference between two population proportions for large and independent samples will be discussed in detail.

8.1 Hypothesis

Definition

Null hypothesis is a hypothesis which is tested for possible rejection under the assumption that it is true and is denoted by H_0

Alternative hypothesis is a complimentary hypothesis to null hypothesis and is denoted by H_1

8.2 Procedures of conducting the tests of hypotheses

Step 1 Formulate H_0 : $\theta_1 \leq \theta_2$, $\theta_1 \geq \theta_2$, $\theta_1 = \theta_2$

Step 2 Formulate H_1 : $\theta_1 > \theta_2$, $\theta_1 < \theta_2$, $\theta_1 \neq \theta_2$

Step 3 Specify α

Step 4 Determine a critical region of size α. (If the conclusion is to be based on a *P*-value, it is not necessary to state the critical region)

Step 5 Compute the value of the test statistic from the sample data

Step 6 Conclusions : Reject H_0 if the statistic has a value in the critical region (or if the computed *P*-value is less than or equal to the desired significance level α); otherwise, do not reject H_0.

8.3 Two sample: Test concerning Difference Between Two Means

Case 1 : "*σ_1^2 and σ_2^2 are known*" *or*

"σ_1^2 *and* σ_2^2 *are unknown but* $n_1, n_2 \geq 30$ "

Two independent samples of size n_1 and n_2 taken from population with mean μ_1 , μ_2 and variance σ_1^2 *and* σ_2^2. To test whether is these samples are taken from population whose means are equal,

For two-tailed hypothesis,

(i) $H_0 : \mu_1 = \mu_2$ / $H_0 : \mu_1 - \mu_2 = 0$

(ii) $H_1 : \mu_1 \neq \mu_2$

(iii) $\alpha = 0.05$

(iv) Critical region : Reject H_0 if $Z > Z_{\alpha/2}$ *or* $Z < -Z_{\alpha/2}$

(v) Test statistic $$Z = \frac{(\overline{x_1} - \overline{x_2}) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

- **Note :** ***Replace*** σ_1 ***and*** σ_2 ***with*** s_1 ***and*** s_2 ***if*** σ_1 ***and*** σ_2 ***are unknown***

Case 2 : **"σ_1^2 *and* σ_2^2 *are unknown ,* $n_1, \; n_2 \leq 30$ "**

Two independent samples of size n_1 and n_2 taken from approximate normal population with mean μ_1 , μ_2 and unknown variances. To test whether is these samples are taken from population whose means are equal,

Case 2.1 : $\sigma_1^2 = \sigma_2^2$ ***(Equal variance)***

For two-tailed hypothesis,

(i) $H_0 : \mu_1 = \mu_2$

(ii) $H_1 : \mu_1 \neq \mu_2$

(iii) $\alpha = 0.05$

(iv) Critical region : Reject H_0 if $T > t_{\alpha/2,\ n_1+n_2-2}$ *or* $T < -t_{\alpha/2,\ n_1+n_2-2}$

(v) Test statistic $$T = \frac{(\overline{x_1} - \overline{x_2}) \;-\; (\mu_1 - \mu_2)}{S_p\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$$

where

$$\textbf{\textit{pooled variance}},\ S_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}$$

Case 2.2 : $\sigma_1^2 \neq \sigma_2^2$ ***(Unequal variance)***

For two-tailed hypothesis,

(iv) $H_0:\ \mu_1 = \mu_2$
(v) $H_1:\ \mu_1 \neq \mu_2$
(vi) $\alpha = 0.05$
(iv) Critical region : Reject H_0 if $T > t_{\alpha/2,\ \nu}$ *or* $T < -t_{\alpha/2,\ \nu}$

(v) Test statistic $$T = \frac{(\overline{x_1} - \overline{x_2}) \;-\; (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

where

$$w_1 = \frac{s_1^2}{n_1}, \qquad w_2 = \frac{s_2^2}{n_2}, \qquad \nu = \frac{(w_1 + w_2)^2}{\dfrac{w_1^2}{n_1 - 1} + \dfrac{w_2^2}{n_2 - 1}}$$

8.4 Two Samples : Test on Two proportions

If $\hat{p}_1$ and $\hat{p}_2$ are the proportion of successes in a random sample of size n_1 and n_2, to test whether the two population proportions are equal,

For two-tailed hypothesis,

(i) $H_0:\ p_1 = p_2$
(ii) $H_1:\ p_1 \neq p_2$
(iii) $\alpha = 0.05$
(iv) Critical region : Reject H_0 if $Z > Z_{\alpha/2}$ *or* $Z < -Z_{\alpha/2}$

(v) Test statistic

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\hat{p}_{pool}\hat{q}_{pool}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where

$$\hat{p}_{pool} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$$