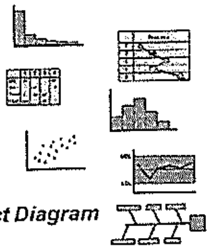
Seven Quality Control Tools

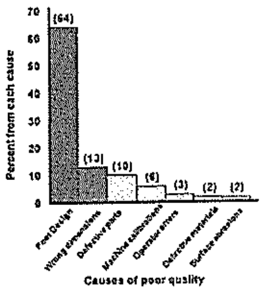
- ✓ *Pareto Analysis*
- ✓ *Flow Chart*
- ✓ *Check Sheet*
- ✓ *Histogram*
- ✓ *Scatter Diagram*
- ✓ *SPC Chart*
- ✓ *Cause-and-Effect Diagram*



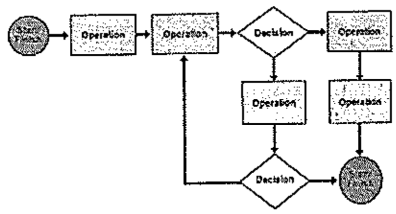
Pareto Chart

CAUSE	NUMBER OF DEFECTS	PERCENTAGE
Poor design	80	64 %
Wrong part dimensions	16	13
Defective parts	12	10
Incorrect machine calibration	7	6
Operator errors	4	3
Defective material	3	2
Surface abrasions	3	2
	125	100 %

Pareto Chart



Flow Chart



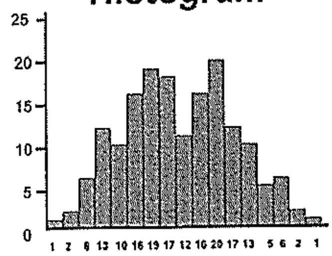
Check Sheet

COMPONENTS REPLACED BY LAB
TIME PERIOD: 22 Feb to 27 Feb 2002
REPAIR TECHNICIAN: Bob

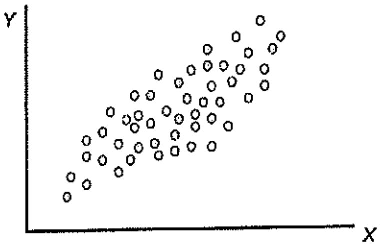
TV SET MODEL 1013

Integrated Circuits	|||||
Capacitors	||||| ||||| ||||| ||||| ||||| ||
Resistors	||
Transformers	|||||
Commands	
CRT		

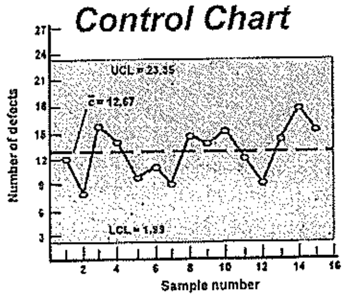
Histogram



Scatter Diagram



Control Chart



Cause-and-Effect Diagram

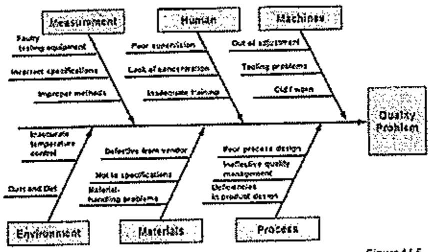


Figure 14.6

Fundamentals of Hypothesis Testing:

One Sample Tests
Population Mean

The Hypothesis

- A hypothesis is a claim (assumption) about a population parameter:
  - population mean

    Example: The mean monthly cell phone bill of this city is $\mu = \$52$

The Null Hypothesis, H_0

- States the assumption (numerical) to be tested

  Example: The mean number of TV sets in U.S. Homes is equal to three.

  $$H_0 : \mu = 3$$

- Is always about a population parameter, not about a sample statistic.

The Null Hypothesis, H_0

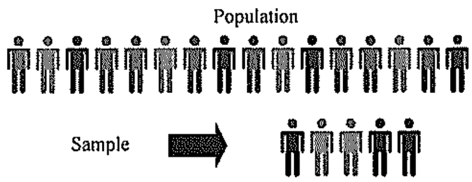
- Begin with the assumption that the null hypothesis is true
  - Similar to the notion of innocent until proven guilty
- It refers to the status quo
- Always contains "=", "≤" or "≥" sign
- May or may not be rejected

The Alternative Hypothesis, H_1

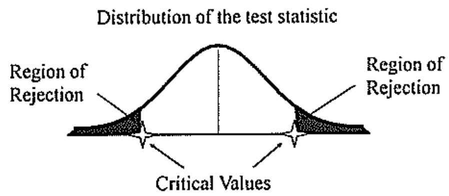
- Is the opposite of the null hypothesis
  - e.g., The mean number of TV sets in U.S. homes is not equal to 3 (H_1: $\mu \neq 3$)
- Contains the "≠", "<" or ">" sign
- May or may not be proven

The Hypothesis Testing Process

- Claim: The population mean age is 50.
  - H_0: $\mu = 50$, H_1: $\mu \neq 50$
- Sample the population and find sample mean.



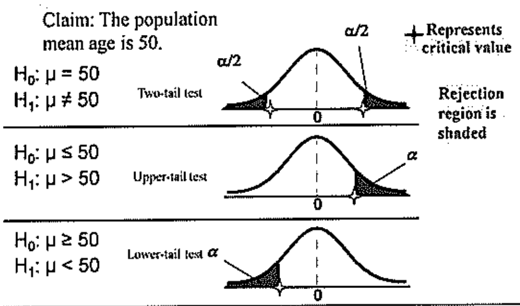
The Test Statistic and Critical Values



Errors in Decision Making

- **Type I Error**
  - Reject a true null hypothesis
  - Considered a serious type of error
  - The probability of a Type I Error is α
    - Called level of significance of the test
    - Set by researcher in advance
- **Type II Error**
  - Failure to reject false null hypothesis
  - The probability of a Type II Error is β

Level of Significance, α



Hypothesis Testing: σ Known

For two tail test for the mean, σ known:

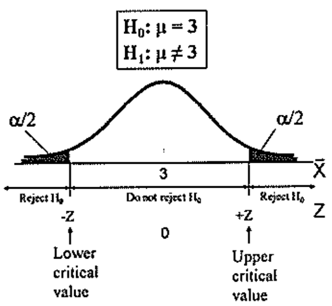
- Convert sample statistic ($\bar{X}$) to test statistic

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

- Determine the critical Z values for a specified level of significance α from a table or by using Excel
- Decision Rule: If the test statistic falls in the rejection region, reject H_0 ; otherwise do not reject H_0

Hypothesis Testing: σ Known

- There are two cutoff values (critical values), defining the regions of rejection



Hypothesis Testing: σ Known

Example: Test the claim that the true mean weight of chocolate bars manufactured in a factory is 3 ounces.

- State the appropriate null and alternative hypotheses
  - H_0: $\mu = 3$ H_1: $\mu \neq 3$ (This is a two tailed test)
- Specify the desired level of significance
  - Suppose that $\alpha = .05$ is chosen for this test
- Choose a sample size
  - Suppose a sample of size n = 100 is selected

Hypothesis Testing: σ Known

- Determine the appropriate technique
  - σ is known so this is a Z test
- Set up the critical values
  - For $\alpha = .05$ the critical Z values are ± 1.96
- Collect the data and compute the test statistic
  - Suppose the sample results are

    $n = 100, \ \bar{X} = 2.84$

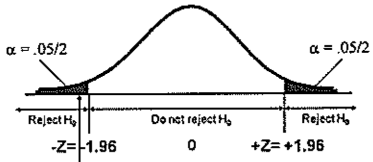
    ($\sigma = 0.8$ is assumed known from past company records)

So the test statistic is: $Z = \dfrac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \dfrac{2.84 - 3}{\frac{0.8}{\sqrt{100}}} = \dfrac{-.16}{.08} = -2.0$

Chap 9-13

Hypothesis Testing: σ Known

- Is the test statistic in the rejection region?



Reject H_0 if $Z < -1.96$ or $Z > 1.96$; otherwise do not reject H_0

Here, $Z = -2.0 < -1.96$, so the test statistic is in the rejection region

Chap 9-14

Hypothesis Testing: σ Known

- Reach a decision and interpret the result
  - Since $Z = -2.0 < -1.96$, you reject the null hypothesis and conclude that there is sufficient evidence that the mean weight of chocolate bars is not equal to 3.

Chap 9-15

Hypothesis Testing: σ Known

6 Steps of Hypothesis Testing:

1. State the null hypothesis, H_0 and state the alternative hypotheses, H_1
2. Choose the level of significance, α, and the sample size n.
3. Determine the appropriate statistical technique and the test statistic to use
4. Find the critical values and determine the rejection region(s)

Chap 9-16

Hypothesis Testing: σ Known

5. Collect data and compute the test statistic from the sample result
6. Compare the test statistic to the critical value to determine whether the test statistic falls in the region of rejection. Make the statistical decision: Reject H_0 if the test statistic falls in the rejection region. Express the decision in the context of the problem

Chap 9-17

Hypothesis Testing: σ Known Confidence Interval Connections

- For $\bar{X} = 2.84$, $\sigma = 0.8$ and $n = 100$, the 95% confidence interval is:

$$2.84 - (1.96)\frac{0.8}{\sqrt{100}} \ \text{ to } \ 2.84 + (1.96)\frac{0.8}{\sqrt{100}}$$

$$2.6832 \le \mu \le 2.9968$$

- Since this interval does not contain the hypothesized mean (3.0), you reject the null hypothesis at $\alpha = .05$

Chap 9-18

Hypothesis Testing: σ Known One Tail Tests

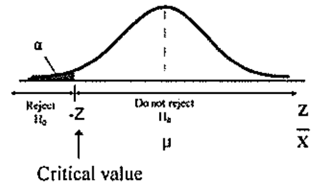
- In many cases, the alternative hypothesis focuses on a particular direction

$H_0: \mu \geq 3$
$H_1: \mu < 3$
→ This is a lower-tail test since the alternative hypothesis is focused on the lower tail below the mean of 3

$H_0: \mu \leq 3$
$H_1: \mu > 3$
→ This is an upper-tail test since the alternative hypothesis is focused on the upper tail above the mean of 3

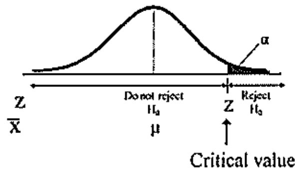
Hypothesis Testing: σ Known Lower Tail Tests

- There is only one critical value, since the rejection area is in only one tail.



Hypothesis Testing: σ Known Upper Tail Tests

- There is only one critical value, since the rejection area is in only one tail.



Hypothesis Testing: σ Known Upper Tail Test Example

A phone industry manager thinks that customer monthly cell phone bills have increased, and now average more than $52 per month. The company wishes to test this claim. Past company records indicate that the standard deviation is about $10.

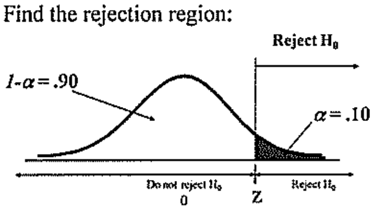
Form hypothesis test:

$H_0: \mu \leq 52$ the mean is less than or equal to than $52 per month

$H_1: \mu > 52$ the mean is greater than $52 per month (i.e., sufficient evidence exists to support the manager's claim)

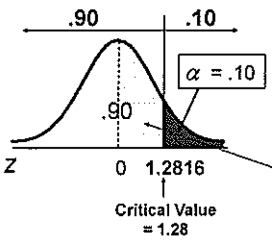
Hypothesis Testing: σ Known Upper Tail Test Example

- Suppose that α = .10 is chosen for this test
- Find the rejection region:

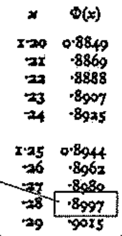


Hypothesis Testing: σ Known Upper Tail Test Example

What is Z given $\alpha = 0.10$?



Standard Normal Distribution Table 4 (Portion)



x	$\Phi(x)$
1·20	0·8849
·21	·8869
·22	·8888
·23	·8907
·24	·8925
1·25	0·8944
·26	·8962
·27	·8980
·28	·8997
·29	·9015

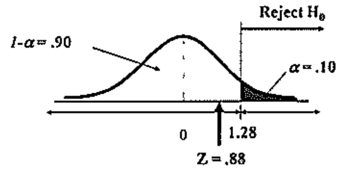
Hypothesis Testing: σ Known Upper Tail Test Example

- Obtain sample and compute the test statistic.
- Suppose a sample is taken with the following results: n = 64, $\bar{X} = 53.1$ (σ=10 was assumed known from past company records)
  - Then the test statistic is:

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{53.1 - 52}{\frac{10}{\sqrt{64}}} = 0.88$$

Hypothesis Testing: σ Known Upper Tail Test Example

- Reach a decision and interpret the result:



Do not reject H_0 since $Z = 0.88 \le 1.28$

i.e.: there is not sufficient evidence that the mean bill is greater than $52

Hypothesis Testing: σ Unknown

- If the population standard deviation is unknown, you instead use the sample standard deviation S.
- Because of this change, you use the t distribution instead of the Z distribution to test the null hypothesis about the mean.
- All other steps, concepts, and conclusions are the same.

Hypothesis Testing: σ Unknown

- Recall that the t test statistic with n-1 degrees of freedom is:

$$t_{n-1} = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}}$$

Hypothesis Testing: σ Unknown Example

The mean cost of a hotel room in New York is said to be $168 per night. A random sample of 25 hotels resulted in $\bar{X}$ = $172.50 and S = 15.40. Test at the $\alpha = 0.05$ level.

(A stem-and-leaf display and a normal probability plot indicate the data are approximately normally distributed)

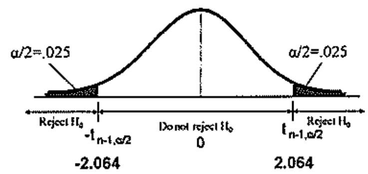
H_0: μ = 168
H_1: μ ≠ 168

Hypothesis Testing: σ Unknown Example

H_0: μ = 168
H_1: μ ≠ 168

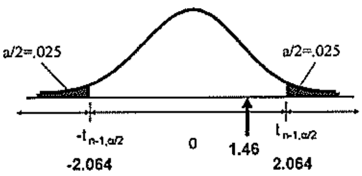
- α = 0.05
- n = 25
- **σ is unknown, so use a t statistic**
- **Critical Value:** $t_{24} = \pm 2.064$

Determine the regions of rejection



Hypothesis Testing: σ Unknown Example

$$t_{n-1} = \frac{\overline{X}-\mu}{\frac{S}{\sqrt{n}}} = \frac{172.50-168}{\frac{15.40}{\sqrt{25}}} = 1.46$$



Do not reject H_0: not sufficient evidence that true mean cost is different from $168

Two-Sample Tests for Population Mean

Two Sample Tests Overview

Two Sample Tests

Independent Population Means

Examples:

Group 1 vs. Group 2

Two Sample Tests

Independent Population Means *

- σ_1 and σ_2 known
- σ_1 and σ_2 unknown

Goal: Test hypothesis or form a confidence interval for the difference between two population means, $\mu_1 - \mu_2$

The point estimate for the difference is

$$\bar{X}_1 - \bar{X}_2$$

Two-Sample Tests Independent Populations

Independent Population Means *

- σ_1 and σ_2 known → Use a Z test statistic
- σ_1 and σ_2 unknown → Use S to estimate unknown σ, use a t test statistic

σ_1 and σ_2 Known

Independent Population means

- σ_1 and σ_2 known *
- σ_1 and σ_2 unknown

Assumptions:

- Samples are randomly and independently drawn
- Population distributions are normal

σ_1 and σ_2 Known

Independent Population means

- σ_1 and σ_2 known *
- σ_1 and σ_2 unknown

When σ_1 and σ_2 are known and both populations are normal or both sample sizes are at least 30, the test statistic is a Z-value...

...and the standard error of $\bar{X}_1 - \bar{X}_2$ is

$$\sigma_{\bar{X}_1-\bar{X}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

σ_1 and σ_2 Known

Independent Population Means

- σ_1 and σ_2 known *
- σ_1 and σ_2 unknown

The test statistic for $\mu_1 - \mu_2$ is:

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Two-Sample Tests Independent Populations

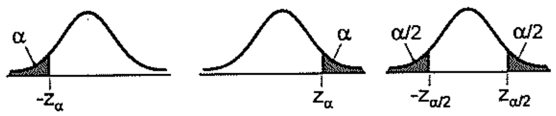
Two Independent Population, Comparing Means

Lower-tail test:	Upper-tail test:	Two-tail test:
$H_0: \mu_1 \ge \mu_2$	$H_0: \mu_1 \le \mu_2$	$H_0: \mu_1 = \mu_2$
$H_1: \mu_1 < \mu_2$	$H_1: \mu_1 > \mu_2$	$H_1: \mu_1 \ne \mu_2$
i.e.,	i.e.,	i.e.,
$H_0: \mu_1 - \mu_2 \ge 0$	$H_0: \mu_1 - \mu_2 \le 0$	$H_0: \mu_1 - \mu_2 = 0$
$H_1: \mu_1 - \mu_2 < 0$	$H_1: \mu_1 - \mu_2 > 0$	$H_1: \mu_1 - \mu_2 \ne 0$

Two-Sample Tests Independent Populations

Two Independent Population, Comparing Means

Lower-tail test:	Upper-tail test:	Two-tail test:
$H_0: \mu_1 - \mu_2 \ge 0$	$H_0: \mu_1 - \mu_2 \le 0$	$H_0: \mu_1 - \mu_2 = 0$
$H_1: \mu_1 - \mu_2 < 0$	$H_1: \mu_1 - \mu_2 > 0$	$H_1: \mu_1 - \mu_2 \ne 0$



Reject H_0 if $Z < -Z_\alpha$ | Reject H_0 if $Z > Z_\alpha$ | Reject H_0 if $Z < -Z_{\alpha/2}$ or $Z > Z_{\alpha/2}$

Worked Example 1

Two types of chocolate chips are suitable for use in decorating cakes. The melting points of these chocolate chips are important. It is known that $\sigma_1 = \sigma_2 = 1.0°C$. From a random sample of size $n_1 = 12$ and $n_2 = 15$, we obtain $\bar{x}_1 = 50°C$ and $\bar{x}_2 = 40°C$. The bakery will use chocolate chip 1 if its mean melting point exceeds that of chocolate chip 2 by at least 12°C. Based on the sample information, should the bakery use chocolate chip 1? Use $\alpha = 0.05$ to make the decision

<u>Solution:</u>

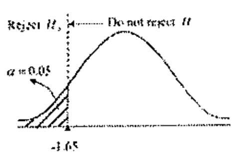
$\sigma_1 = \sigma_2 = 1.0°C$

$n_1 = 12$ $\quad$ $n_2 = 15$

$\bar{x}_1 = 50$ $\quad$ $\bar{x}_2 = 40$

$H_0: \mu_1 - \mu_2 \ge 12$

$H_1: \mu_1 - \mu_2 < 12$



Since σ_1 and σ_2 are known, we use z

$$s_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{1}{12} + \frac{1}{15}} = 0.387298$$

From the normal distribution table, at $\alpha = 0.05$, $z = -1.65$

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_{\bar{x}_1 - \bar{x}_2}} = \frac{(50 - 40) - 12}{0.387298} = -5.16$$

The value of the test statistic $z = -5.16$ is smaller than the critical value of $z = -1.65$ and it falls in the rejection region. As a result, we *reject* H_0 at $\alpha = 0.05$. Therefore, the mean melting point of chocolate chip 1 does not exceed chocolate chip 2 by at least 12°C. The bakery should not use chocolate chip 1

σ_1 and σ_2 Unknown

Independent Population Means

- σ_1 and σ_2 known
- σ_1 and σ_2 unknown *

Assumptions:

- Samples are randomly and independently drawn
- Populations are normally distributed or both sample sizes are at least 30
- Population variances are unknown but assumed equal

σ_1 and σ_2 Unknown

- Independent Population Means
  - σ_1 and σ_2 known
  - σ_1 and σ_2 unknown *

Forming interval estimates:

- The population variances are assumed equal, so use the two sample standard deviations and pool them to estimate σ
- the test statistic is a t value with $(n_1 + n_2 - 2)$ degrees of freedom

σ_1 and σ_2 Unknown

- Independent Population Means
  - σ_1 and σ_2 known
  - σ_1 and σ_2 unknown *

The pooled standard deviation is

$$S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{(n_1-1)+(n_2-1)}}$$

σ_1 and σ_2 Unknown

(continued)

- Independent Population Means
  - σ_1 and σ_2 known
  - σ_1 and σ_2 unknown *

The test statistic for $\mu_1 - \mu_2$ is:

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

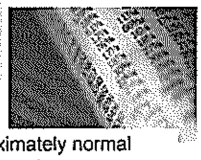
Where t has $(n_1 + n_2 - 2)$ d.f., and

$$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{(n_1-1)+(n_2-1)}$$

Pooled Variance t Test: Example

You are a financial analyst for a brokerage firm. Is there a difference in dividend yield between stocks listed on the NYSE & NASDAQ? You collect the following data:

	NYSE	NASDAQ
Number	21	25
Sample mean	3.27	2.53
Sample std dev	1.30	1.16



Assuming both populations are approximately normal with equal variances, is there a difference in average yield (α = 0.05)?

Calculating the Test Statistic

The test statistic is:

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(3.27 - 2.53) - 0}{\sqrt{1.5021\left(\frac{1}{21} + \frac{1}{25}\right)}} = \boxed{2.040}$$

$$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{(n_1-1)+(n_2-1)} = \frac{(21-1)1.30^2 + (25-1)1.16^2}{(21-1)+(25-1)} = 1.5021$$

Solution

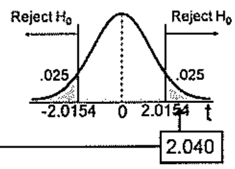
H_0: $\mu_1 - \mu_2 = 0$ i.e. $(\mu_1 = \mu_2)$

H_1: $\mu_1 - \mu_2 \neq 0$ i.e. $(\mu_1 \neq \mu_2)$

α = 0.05

df = 21 + 25 - 2 = 44

Critical Values: t = ± 2.0154



Test Statistic:

$$t = \frac{3.27 - 2.53}{\sqrt{1.5021\left(\frac{1}{21} + \frac{1}{25}\right)}} = \boxed{2.040}$$

Decision:
Reject H_0 at α = 0.05

Conclusion:
There is evidence of a difference in means.

Example 3: Mean for Two-Match Sample t-test

Data on knowledge level were collected both before and after a training programme. Test for the difference in knowledge level at .05 level of significance. State the appropriate null and alternative hypothesis, and your conclusion.

	Pre-score	Post score	Difference (d)
	4	8	4
	5	6	1
	3	5	2
	4	7	3
	6	9	3
	3	6	3
	2	5	3
	5	6	1
	4	6	2
	3	7	4
Total	39	65	26
Mean	3.9	6.5	2.6
S.D.	1.197	1.269	1.075

SOLUTION:

The hypotheses: ← Step 1

H_0: $\mu_d = 0$

H_A: $\mu_d \neq 0$

Summary data: ← Step 2

$\mu_d = 0$ $\quad s_d = 1.075$

$\bar{d} = 2.6$ $\quad n = 10$

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} = \frac{2.6 - 0}{\frac{1.075}{\sqrt{10}}} = \frac{2.6}{.3399} = 7.649$$

$df = 10 - 1 = 9$

$t_{\frac{.05}{2}, 9} = \pm 2.262$ ← Step 3

Since t_{cal} is bigger than t_{table} Reject H_0 ← Step 4

Step 5 → Conclude there is a significant difference in the knowledge scores between the pre and post training.

T-Test

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	POST	6.5000	10	[illegible]	.4014
	PRE	3.9000	10	1.1972	[illegible]

Paired Samples Correlations

		N	Correlation	Sig.
Pair 1	POST & PRE	10	[illegible]	[illegible]

Paired Samples Test

			Pair 1 POST - PRE
Paired Differences	Mean		2.6000
	Std. Deviation		1.0750
	Std. Error Mean		.3399
	95% Confidence Interval of the Difference	Lower	1.8310
		Upper	3.3690
t			7.649
df			9
Sig. (2-tailed)			.000

Example 11: Analysis of Variance

Given the following data on refer to time (in minutes) taken to complete a given task among three worker groups.

GROUP 1	GROUP 2	GROUP 3
14	9	11
12	10	12
11	8	13
13	9	10
12	8	13
13	11	14
11	8	12
10	9	11

Based on the above data set:

1. Test the hypothesis that three is a significant difference in time taken among the three worker groups to complete a given task at .05 level of significance.

SOLUTION 1:

Before you respond to the test, calculate the following summary statistics. These statistics will be used in the subsequent calculations. The values for the summary statistics follows:

	GROUP 1	GROUP 2	GROUP 3	Total
Sample size	8	8	8	24
Mean	12	9	12	11
ΣX	96	72	96	264
ΣX^2	1,164	656	1,164	2,984

Step 1: Hypotheses

H_0: $\mu_1 = \mu_2 = \mu_3$

H_A: Not all means are equal

OR

$H_0: \sigma_B^2 = \sigma_w^2$

$H_A: \sigma_B^2 \neq \sigma_w^2$

Step 2: Calculate Test Statistic

1. Three sum of squares

$$SST = \Sigma X^2 - \frac{(\Sigma X)^2}{N}$$
$$= 2{,}984 - \frac{264^2}{24}$$
$$= 2{,}954 - 2{,}904$$
$$= 80$$

$$SSB = \sum \frac{T_j^2}{n_j} - \frac{(\Sigma X)^2}{N}$$
$$= \left(\frac{96^2}{8} + \frac{72^2}{8} + \frac{96^2}{8}\right) - \frac{264^2}{24}$$
$$= 2{,}952 - 2{,}904$$
$$= 48$$

$$SSW = SST - SSB$$
$$= 80 - 48$$
$$= 32$$

2. Degrees of freedom

$$df_B = k - 1$$
$$= 3 - 1$$
$$= 2$$
$$df_W = N - k$$
$$= 24 - 3$$
$$= 21$$
$$df_T = N - 1$$
$$= 24 - 1$$
$$= 23$$

3. Summary ANOVA table

Source	SS	df	MS	F
Between group	48	2	24	15.75
Within group	32	21	1.524	
TOTAL	80	23		

Step 3: Determine critical value

$F_{21}^{2}(.05) = 3.49$

Step 4: Decision

Since $F_{cal} > F_{critical}$

∴ Reject the null hypothesis

Step 5: Conclusion

Conclude that there is significant difference in worker groups at .05 level of significance.

F Distribution Alpha = .05

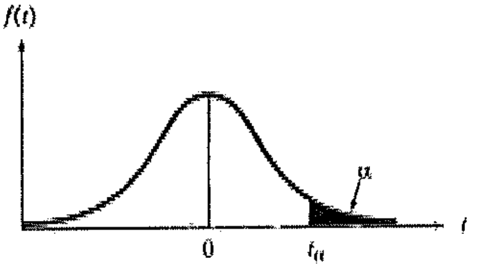
Degrees of Freedom for Denominator	Degrees of Freedom for Numerator: 1	2	3	4	5	6	7	8	9	10	15	20	25	30	40	50
1	161.4	199.5	215.8	224.8	230.0	233.8	236.5	238.6	240.1	242.1	245.2	248.4	248.9	250.5	250.8	252.6
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.43	19.44	19.46	19.47	19.48	19.48
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.70	8.66	8.63	8.62	8.59	8.58
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.86	5.80	5.77	5.75	5.72	5.70
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.62	4.56	4.52	4.50	4.46	4.44
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	3.94	3.87	3.83	3.81	3.77	3.75
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.51	3.44	3.40	3.38	3.34	3.32
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.22	3.15	3.11	3.08	3.04	3.02
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.01	2.94	2.89	2.86	2.83	2.80
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.85	2.77	2.73	2.70	2.66	2.64
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.72	2.65	2.60	2.57	2.53	2.51
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.62	2.54	2.50	2.47	2.43	2.40
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.53	2.46	2.41	2.38	2.34	2.31
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.46	2.39	2.34	2.31	2.27	2.24
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.40	2.33	2.28	2.25	2.20	2.18
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.35	2.28	2.23	2.19	2.15	2.12
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.31	2.23	2.18	2.15	2.10	2.08
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.27	2.19	2.14	2.11	2.06	2.04
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.23	2.16	2.11	2.07	2.03	2.00
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.20	2.12	2.07	2.04	1.99	1.97
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.15	2.07	2.02	1.98	1.94	1.91
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.11	2.03	1.97	1.94	1.89	1.86
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.07	1.99	1.94	1.90	1.85	1.82
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.04	1.96	1.91	1.87	1.82	1.79
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.01	1.93	1.88	1.84	1.79	1.76
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	1.92	1.84	1.78	1.74	1.69	1.66
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.87	1.78	1.73	1.69	1.63	1.60
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.84	1.75	1.69	1.65	1.59	1.56
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.75	1.66	1.60	1.55	1.50	1.46
200	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.72	1.62	1.56	1.52	1.46	1.41
500	3.86	3.01	2.62	2.39	2.23	2.12	2.03	1.96	1.90	1.85	1.69	1.59	1.53	1.48	1.42	1.38
1000	3.85	3.01	2.61	2.38	2.22	2.11	2.02	1.95	1.89	1.84	1.68	1.58	1.52	1.47	1.41	1.36

F Distribution Alpha +.01

Degrees of Freedom for Denominator	Degrees of Freedom for Numerator															
	1	2	3	4	5	6	7	8	9	10	15	20	25	30	40	50
1	4048	4993	5377	5577	5668	5924	5992	6096	6132	6168	6079	6168	6214	6355	6168	6213
2	98.50	99.01	99.15	99.23	99.30	99.33	99.35	99.39	99.40	99.43	99.38	99.48	99.43	99.37	99.44	99.59
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.34	27.23	26.87	26.69	26.58	26.51	26.41	26.36
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.20	14.02	13.91	13.84	13.75	13.69
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.72	9.55	9.45	9.38	9.29	9.24
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.56	7.40	7.30	7.23	7.14	7.09
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.31	6.16	6.06	5.99	5.91	5.86
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.52	5.36	5.26	5.20	5.12	5.07
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	4.96	4.81	4.71	4.65	4.57	4.52
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.56	4.41	4.31	4.25	4.17	4.12
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.25	4.10	4.01	3.94	3.86	3.81
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.01	3.86	3.76	3.70	3.62	3.57
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.82	3.66	3.57	3.51	3.43	3.38
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.66	3.51	3.41	3.35	3.27	3.22
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.52	3.37	3.28	3.21	3.13	3.08
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.41	3.26	3.16	3.10	3.02	2.97
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.31	3.16	3.07	3.00	2.92	2.87
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.23	3.08	2.98	2.92	2.84	2.78
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.15	3.00	2.91	2.84	2.76	2.71
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.09	2.94	2.84	2.78	2.69	2.64
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	2.98	2.83	2.73	2.67	2.58	2.53
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	2.89	2.74	2.64	2.58	2.49	2.44
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.81	2.66	2.57	2.50	2.42	2.36
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.75	2.60	2.51	2.44	2.35	2.30
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.70	2.55	2.45	2.39	2.30	2.25
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.52	2.37	2.27	2.20	2.11	2.06
50	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78	2.70	2.42	2.27	2.17	2.10	2.01	1.95
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.35	2.20	2.10	2.03	1.94	1.88
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.19	2.03	1.93	1.86	1.76	1.70
200	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50	2.41	2.13	1.97	1.87	1.79	1.69	1.63
500	6.69	4.65	3.82	3.36	3.05	2.84	2.68	2.55	2.44	2.36	2.07	1.92	1.81	1.74	1.63	1.57
1000	6.67	4.63	3.80	3.34	3.04	2.82	2.66	2.53	2.43	2.34	2.06	1.90	1.79	1.72	1.61	1.54

Source: The entries in this table were computed by the author

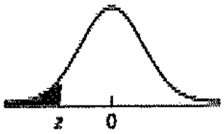
TABLE VI Critical Values of t



Degrees of Freedom	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	$t_{.001}$	$t_{.0005}$
1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Source: This table is reproduced with the kind permission of the Trustees of Biometrika from E. S. Pearson and H. O. Hartley (eds.), *The Biometrika Tables for Statisticians*, Vol. 1, 3d ed., Biometrika, 1966.

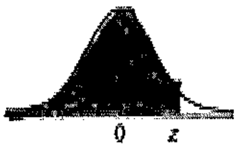
TABLE II
Areas under the standard normal curve



Second decimal place in z										
0.09	*0.08*	*0.07*	*0.06*	*0.05*	*0.04*	*0.03*	*0.02*	*0.01*	*0.00*	*z*
									0.0000[†]	−3.9
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	−3.8
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	−3.7
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	−3.6
0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	−3.5
0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	−3.4
0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005	−3.3
0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007	−3.2
0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010	−3.1
0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013	−3.0
0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019	−2.9
0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026	−2.8
0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035	−2.7
0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047	−2.6
0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062	−2.5
0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082	−2.4
0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107	−2.3
0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139	−2.2
0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179	−2.1
0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228	−2.0
0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287	−1.9
0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359	−1.8
0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446	−1.7
0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548	−1.6
0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668	−1.5
0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808	−1.4
0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968	−1.3
0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151	−1.2
0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357	−1.1
0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587	−1.0
0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841	−0.9
0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119	−0.8
0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420	−0.7
0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743	−0.6
0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085	−0.5
0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446	−0.4
0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821	−0.3
0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207	−0.2
0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602	−0.1
0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000	−0.0

[†] For $z \leq -3.90$, the areas are 0.0000 to four decimal places.

TABLE II (cont.)
Areas under the standard normal curve



	Second decimal place in *z*									
z	*0.00*	*0.01*	*0.02*	*0.03*	*0.04*	*0.05*	*0.06*	*0.07*	*0.08*	*0.09*
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000†									

† For $z \geq 3.90$, the areas are 1.0000 to four decimal places.

Control Charts for Variables

EBB 341 Quality Control

Variation

- There is no two natural items in any category are the same.
- Variation may be quite large or very small.
- If variation very small, it may appear that items are identical, but precision instruments will show differences.

Source of variation

- Equipment
  - *Tool wear, machine vibration, …*
- Material
  - *Raw material quality*
- Environment
  - *Temperature, pressure, humadity*
- Operator
  - *Operator performs- physical & emotional*

Control Chart Viewpoint

- Variation due to
  - Common or chance causes
  - Assignable causes
- Control chart may be used to discover "assignable causes"

Some Terms

- Run chart - without any upper/lower limits
- Specification/tolerance limits - not statistical
- Control limits - statistical

Control chart functions

- Control charts are powerful aids to understanding the performance of a process over time.

What's causing variability?

Control charts identify variation

- Chance causes - "common cause"
  - inherent to the process or random and not controllable
  - if only common cause present, the process is considered stable or "in control"
- Assignable causes - "special cause"
  - variation due to outside influences
  - if present, the process is "out of control"

Control charts help us learn more about processes

- Separate common and special causes of variation
- Determine whether a process is in a state of statistical control or out-of-control
- Estimate the process parameters (mean, variation) and assess the performance of a process or its capability

Control charts to monitor processes

- To monitor output, we use a control chart
  - we check things like the mean, range, standard deviation
- To monitor a process, we typically use **two** control charts
  - mean (or some other central tendency measure)
  - variation (typically using range or standard deviation)

Types of Data

- Variable data
  - Product characteristic that can be measured
    - Length, size, weight, height, time, velocity
- Attribute data
  - Product characteristic evaluated with a discrete choice
    - Good/bad, yes/no

Control chart for <u>variables</u>

- Variables are the <u>measurable</u> characteristics of a product or service.
- Measurement data is taken and arrayed on charts.

Control charts for <u>variables</u>

- **X-bar chart**
  - In this chart the sample *means* are plotted in order to control the mean value of a variable (e.g., size of piston rings, strength of materials, etc.).
- **R chart**
  - In this chart, the sample *ranges* are plotted in order to control the variability of a variable.
- **S chart**
  - In this chart, the sample *standard deviations* are plotted in order to control the variability of a variable.
- **S^2 chart**
  - In this chart, the sample *variances* are plotted in order to control the variability of a variable.

X-bar and R charts

- The X- bar chart is developed from the average of each subgroup data.
  - used to detect changes in the mean between subgroups.
- The R- chart is developed from the ranges of each subgroup data
  - used to detect changes in variation within subgroups

Control chart components

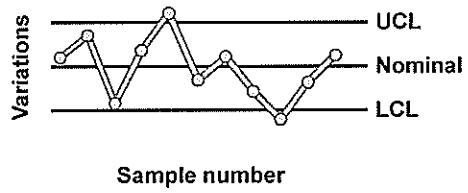
- Centerline
  - shows where the process average is centered or the central tendency of the data
- Upper control limit (UCL) and Lower control limit (LCL)
  - describes the process spread

The Control Chart Method

X bar Control Chart:
UCL = XDmean + A2 x Rmean
LCL = XDmean - A2 x Rmean
CL = XDmean

R Control Chart:
UCL = D4 x Rmean
LCL = D3 x Rmean
CL = Rmean
Capability Study:
PCR = (USL - LSL)/(6s); where s = Rmean /d2

Control Chart Examples



How to develop a control chart?

Define the problem

- Use other quality tools to help determine the general problem that's occurring and the process that's suspected of causing it.

Select a quality characteristic to be measured
- Identify a characteristic to study - for example, part length or any other variable affecting performance.

Choose a subgroup size to be sampled

- Choose homogeneous subgroups
  - Homogeneous subgroups are produced under the same conditions, by the same machine, the same operator, the same mold, at approximately the same time.
- Try to maximize chance to detect differences between subgroups, while minimizing chance for difference with a group.

Collect the data

- Generally, collect 20-25 subgroups (100 total samples) before calculating the control limits.
- Each time a subgroup of sample size n is taken, an average is calculated for the subgroup and plotted on the control chart.

Determine trial centerline

- The centerline should be the population mean, μ
- Since it is unknown, we use X Double bar, or the grand average of the subgroup averages.

$$\overline{\overline{X}} = \frac{\sum_{i=1}^{m} \overline{X}_i}{m}$$

Determine trial control limits - Xbar chart

- The normal curve displays the distribution of the sample averages.
- A control chart is a time-dependent pictorial representation of a normal curve.
- Processes that are considered under control will have 99.73% of their graphed averages fall within 6σ.

UCL & LCL calculation

$$UCL = \overline{\overline{X}} + 3\sigma$$

$$LCL = \overline{\overline{X}} - 3\sigma$$

$$\sigma = \text{standard deviation}$$

Determining an alternative value for the standard deviation

$$\overline{R} = \frac{\sum_{i=1}^{m} R_i}{m}$$

$$UCL = \overline{\overline{X}} + A_2 \overline{R}$$

$$LCL = \overline{\overline{X}} - A_2 \overline{R}$$

Determine trial control limits - R chart

- The range chart shows the spread or dispersion of the individual samples within the subgroup.
  - If the product shows a wide spread, then the individuals within the subgroup are not similar to each other.
  - Equal averages can be deceiving.
- Calculated similar to x-bar charts;
  - Use D_3 and D_4 (appendix 2)

Example: Control Charts for Variable Data

Sample	Slip Ring Diameter (cm) 1	2	3	4	5	$\bar{X}$	R
1	5.02	5.01	4.94	4.99	4.96	4.98	0.08
2	5.01	5.03	5.07	4.95	4.96	5.00	0.12
3	4.99	5.00	4.93	4.92	4.99	4.97	0.08
4	5.03	4.91	5.01	4.98	4.89	4.96	0.14
5	4.95	4.92	5.03	5.05	5.01	4.99	0.13
6	4.97	5.06	5.06	4.96	5.03	5.01	0.10
7	5.05	5.01	5.10	4.96	4.99	5.02	0.14
8	5.09	5.10	5.00	4.99	5.08	5.05	0.11
9	5.14	5.10	4.99	5.08	5.09	5.08	0.15
10	5.01	4.98	5.08	5.07	4.99	5.03	0.10
						50.09	1.15

Calculation

From Table above:

- Sigma X-bar = 50.09
- Sigma R = 1.15
- m = 10

Thus;

- X-Double bar = 50.09/10 = **5.009 cm**
- R-bar = 1.15/10 = **0.115 cm**

Note: The control limits are only preliminary with 10 samples. It is desirable to have at least 25 samples.

Trial control limit

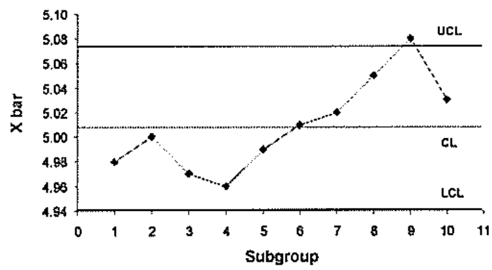
- $UCL_{x\text{-bar}}$ = X-D bar + A_2 R-bar = 5.009 + (0.577)(0.115) = **5.075 cm**
- $LCL_{x\text{-bar}}$ = X-D bar - A_2 R-bar = 5.009 - (0.577)(0.115) = **4.943 cm**
- UCL_R = D_4R-bar = (2.114)(0.115) = **0.243 cm**
- LCL_R = D_3R-bar = (0)(0.115) = **0 cm**

For A_2, D_3, D_4: see Table B, Appendix n = 5

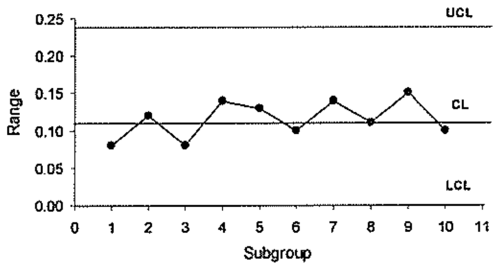
3-Sigma Control Chart Factors

Sample size n	$\bar{X}$-chart A_2	R-chart D_3	D_4
2	1.88	0	3.27
3	1.02	0	2.57
4	0.73	0	2.28
5	0.58	0	2.11
6	0.48	0	2.00
7	0.42	0.08	1.92
8	0.37	0.14	1.86

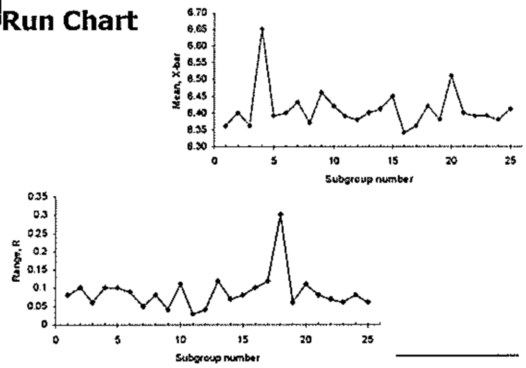
X-bar Chart



R Chart



Run Chart



Revise the charts

- In certain cases, control limits are revised because:
  - out-of-control points were included in the calculation of the control limits.
  - the process is in-control but the within subgroup variation significantly improves.

Revising the charts

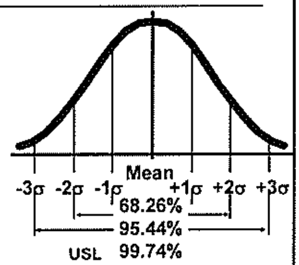
- Interpret the original charts
- Isolate the causes
- Take corrective action
- Revise the chart
  - Only remove points for which you can determine an assignable cause

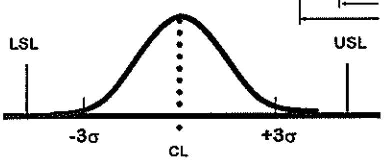
Process in Control

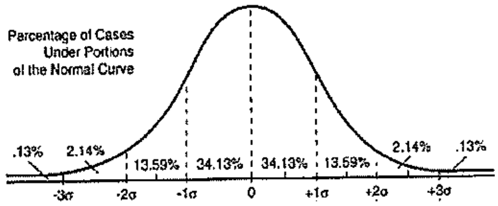
- When a process is in control, there occurs a natural pattern of variation.
- Natural pattern has:
  - About 34% of the plotted point in an imaginary band between 1σ on both side CL.
  - About 13.5% in an imaginary band between 1σ and 2σ on both side CL.
  - About 2.5% of the plotted point in an imaginary band between 2σ and 3σ on both side CL.

The Normal Distribution

σ = Standard deviation





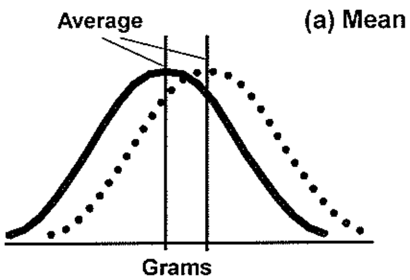


- 34.13% of data lie between μ and 1σ above the mean (μ).
- 34.13% between μ and 1σ below the mean.
- Approximately two-thirds (68.28 %) within 1σ of the mean.
- 13.59% of the data lie between one and two standard deviations
- Finally, almost all of the data (99.74%) are within 3σ of the mean.

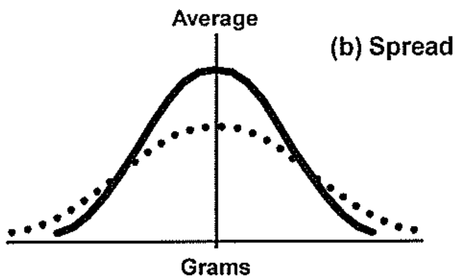
Process Out of Control

- The term out of control is a change in the process due to an assignable cause.
- When a point (subgroup value) falls outside its control limits, the process is out of control.

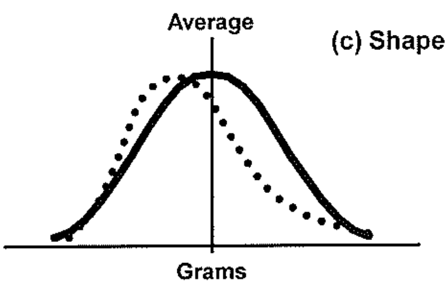
Assignable Causes



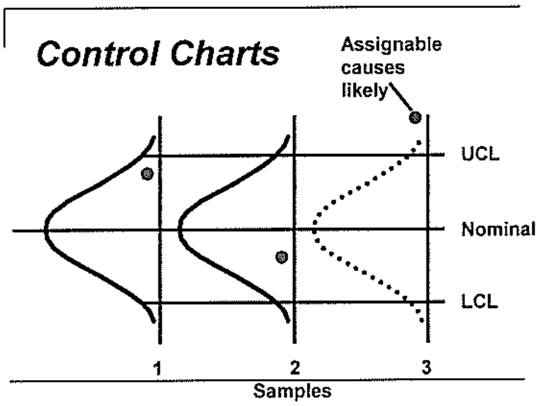
Assignable Causes



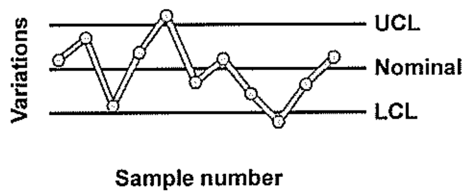
Assignable Causes



Control Charts



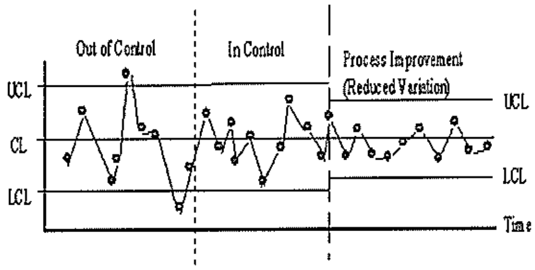
Control Chart Examples



Achieve the purpose

- Our goal is to decrease the variation inherent in a process over time.
- As we improve the process, the spread of the data will continue to decrease.
- Quality improves!!

Improvement



Examine the process

- A process is considered to be stable and in a state of control, or under control, when the performance of the process falls within the statistically calculated control limits and exhibits only chance, or common causes.

TABLE E.11
Control Chart Factors

Number of Observations in Sample	d_2	d_3	D_3	D_4	A_2
2	1.128	0.853	0	3.267	1.880
3	1.693	0.888	0	2.574	1.023
4	2.059	0.880	0	2.282	0.729
5	2.326	0.864	0	2.114	0.577
6	2.534	0.848	0	2.004	0.483
7	2.704	0.833	0.076	1.924	0.419
8	2.847	0.820	0.136	1.864	0.373
9	2.970	0.808	0.184	1.816	0.337
10	3.078	0.797	0.223	1.777	0.308
11	3.173	0.787	0.256	1.744	0.285
12	3.258	0.778	0.283	1.717	0.266
13	3.336	0.770	0.307	1.693	0.249
14	3.407	0.763	0.328	1.672	0.235
15	3.472	0.756	0.347	1.653	0.223
16	3.532	0.750	0.363	1.637	0.212
17	3.588	0.744	0.378	1.622	0.203
18	3.640	0.739	0.391	1.608	0.194
19	3.689	0.734	0.403	1.597	0.187
20	3.735	0.729	0.415	1.585	0.180
21	3.778	0.724	0.425	1.575	0.173
22	3.819	0.720	0.434	1.566	0.167
23	3.858	0.716	0.443	1.557	0.162
24	3.895	0.712	0.451	1.548	0.157
25	3.931	0.708	0.459	1.541	0.153

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